Phénomènes de rotation dans les plasmas partiellement magnétisés

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GDR EMILI, 23-26 octobre 2023

J.P. Boeuf and A. Smolyakov, Physics of Plasmas 30, 050901 (2023)

OUTLINE

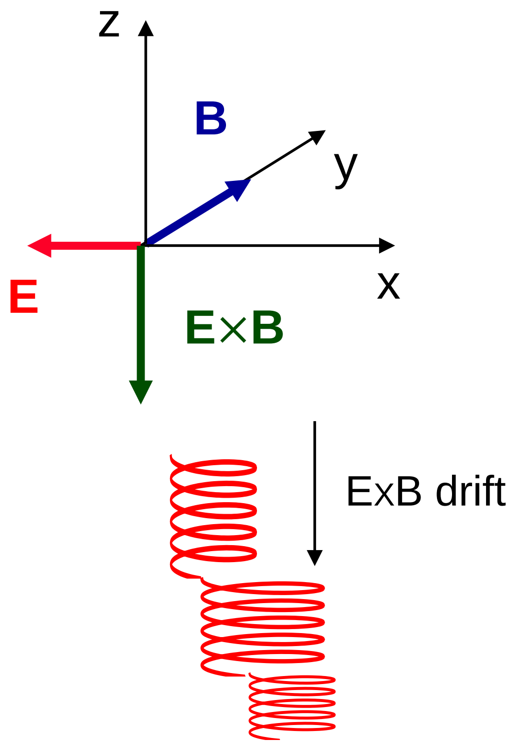
1. **Basic physics of low temperature partially magnetized $\boldsymbol{E} \times \boldsymbol{B}$ plasmas**
   Classical, collisional cross-field transport, Hall parameter
   Conditions and orders of magnitude
   Closed drift devices
   Instabilities and anomalous transport – Dispersion relations

2. **Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharges devices – Two examples**
   Micro-instability in Hall thrusters: the Electron cyclotron drift instability (ECDI)
   Macro-instability in Magnetron discharges and Hall thrusters and evolution toward an ionization instability

3. **Conclusion**

1. Basic physics of low temperature $E \times B$ plasmas

➢ Electron trajectory in E×B fields



- Electron drift in E×B direction $u_{\boldsymbol{e}} = E/B$
- Electrons are trapped in the E direction: no transport in the E direction without collisions
- Free motion along B

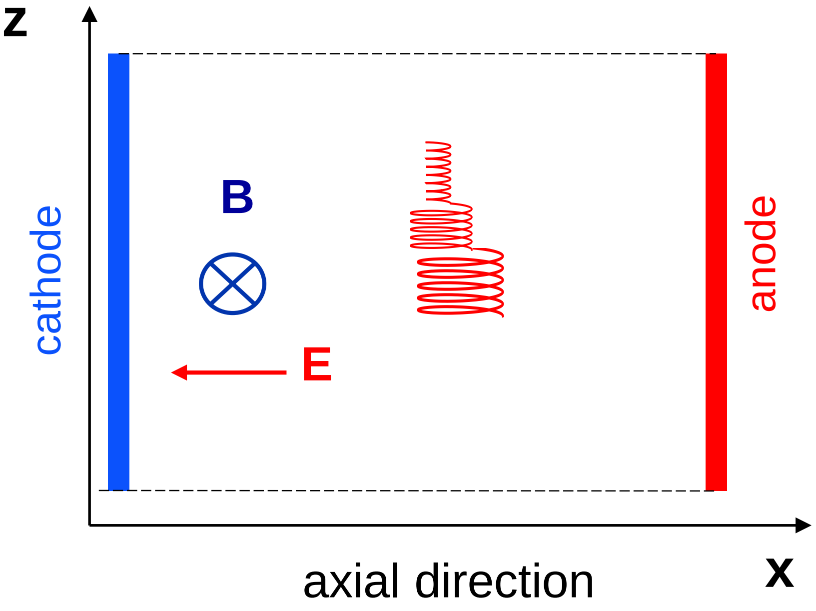
cyclotron frequency $\Omega_{ce} = \dfrac{eB}{m}$

Larmor radius $\rho_e = \dfrac{v_\perp}{\Omega_{ce}}$

- Electron-neutral collisions allow transport // E

1. Basic physics of low temperature $E \times B$ plasmas

- ➢ **Low pressure discharge in small device** (e.g. mtorr, cms)
- ▪ Electron mean free path >> dimensions
- → Magnetic field ⊥ to applied electric field to confine electrons
- → electron residence time is increased allowing ionization and plasma generation
- ▪ Partially magnetized plasmas $\rho_e \ll L$, $\rho_i > L$,



electrons trapped by B
→ collisional electrons

ions ~ not sensitive to B
→ weakly collisional ions

⇩

applications to ion sources

- Plasma processing sputtering / deposition
- Ion thrusters for space propulsion
- Generation of ion beam (eg ITER NBIS)

1. Basic physics of low temperature $\boldsymbol{E \times B}$ plasmas

➢ Conditions and orders of magnitude

B	$10^2 - 10^3$ G
$\Omega_{ce} = eB/m$	$10^9 - 10^{10}$ s^{-1}
$\rho_e = v_e/\Omega_{ce}$	50 – 500 µm
$\Omega_{ci} = eB/M$	$10^4 - 10^5$ s^{-1}
$\rho_i = v_i/\Omega_{ci}$	5 – 50 mm

p	0.1 – 10 Pa
$\nu_{en} \equiv \nu$	$10^6 - 10^8$ s^{-1}
ν_{in}	$10^4 - 10^6$ s^{-1}

Hall parameter h characterizes electron confinement efficiency

$$\boldsymbol{h = \Omega_{ce} / \nu_{en}}$$

p=1 mtorr
B=100 G

$$h \approx 2\ 10^9/2\ 10^6 = 10^3$$

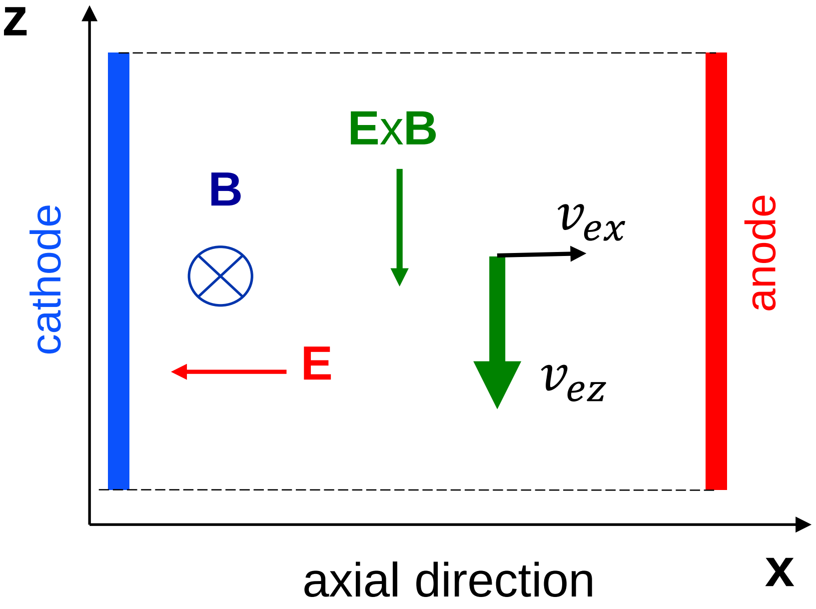
- can be as large as or larger than 10^3
- can vary by several orders of magnitude in the same device

1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Classical, collisional transport theory:

- Simplified electron momentum equation assuming axial electric field and axial plasma density gradient

$$en[\boldsymbol{E} + \boldsymbol{v_e} \times \boldsymbol{B}] + e\nabla(nT_e) + mn\nu_{en}\boldsymbol{v_e} = \boldsymbol{0}$$



$$\mu_{e0} = \frac{e}{m\nu_{en}}$$

$$\left\{ \begin{aligned} v_{ex} &= -\frac{\mu_{e0}}{1+h^2}\left(E_x + \frac{1}{n}\frac{\partial nT_e}{\partial x}\right) \\ v_{ez} &= \mu_{e0}\frac{h}{1+h^2}\left(E_x + \frac{1}{n}\frac{\partial nT_e}{\partial x}\right) \end{aligned} \right\}$$

Hall parameter $h = \frac{eB}{m\nu_{en}} = \frac{\Omega_{ce}}{\nu_{en}} \gg 1$

1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

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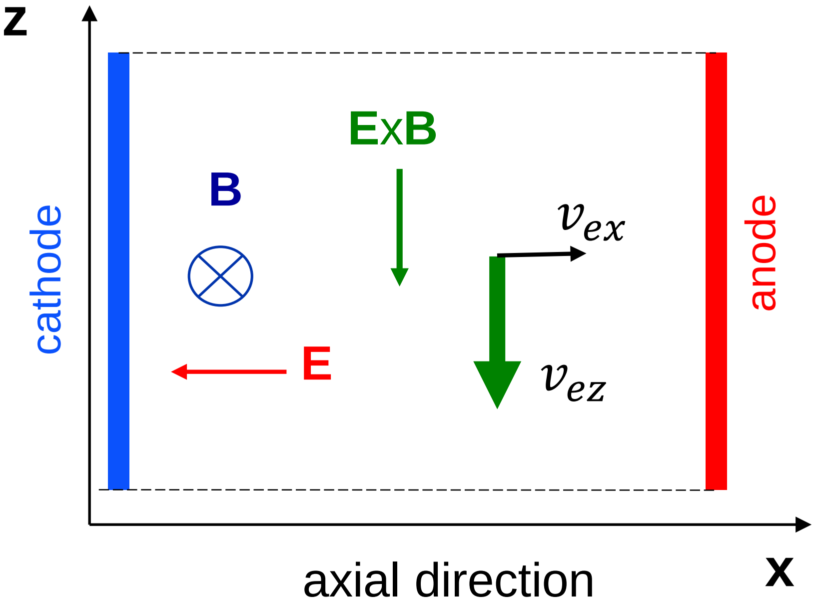

$$\mu_{e0} = \frac{e}{m\nu_{en}}$$

$$\begin{cases} v_{ex} \approx -\dfrac{\mu_{e0}}{h^2}\left(E_x + \dfrac{1}{n}\dfrac{\partial nT_e}{\partial x}\right) \\ v_{ez} \approx \dfrac{1}{B}\left(E_x + \dfrac{1}{n}\dfrac{\partial nT_e}{\partial x}\right) \end{cases}$$

$E \times B$ drift

$\nabla p \times B$ (diagnostic) drift

Hall parameter $h = \dfrac{eB}{m\nu_{en}} = \dfrac{\Omega_{ce}}{\nu_{en}} \gg 1$

1. Basic physics of low temperature $E \times B$ plasmas

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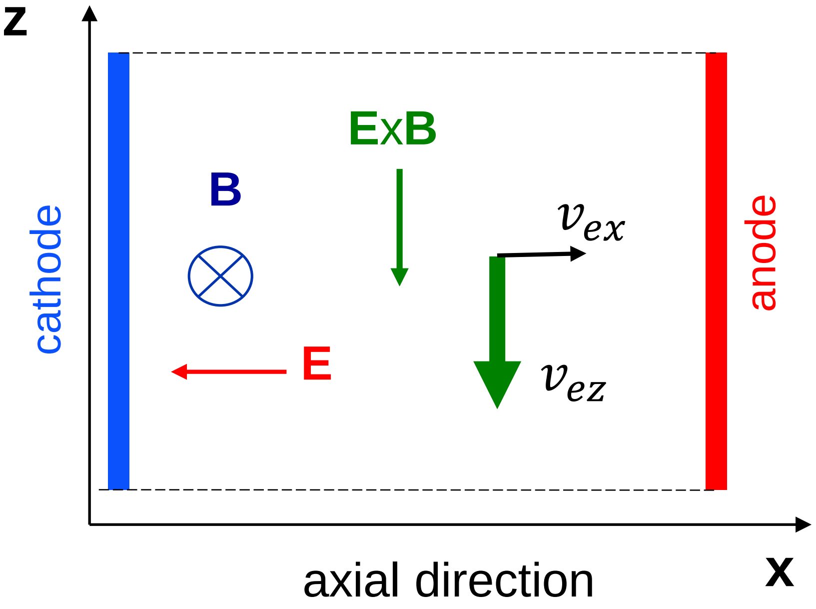


$$\mu_{e0} = \frac{e}{m\nu_{en}}$$

$$\frac{v_{ez}}{v_{ex}} = -\frac{h^2}{\mu_{e0}B} = -h$$

Hall parameter $h = \frac{eB}{m\nu_{en}} = \frac{\Omega_{ce}}{\nu_{en}} \gg 1$

1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Classical, collisional transport theory:

- $\mu_{\parallel E} \approx \frac{\mu_{e0}}{h^2}$. Electron mobility parallel to $\boldsymbol{E}$ is reduced by h^2
- Large drift velocity in $\boldsymbol{E} \times \boldsymbol{B}$ direction (→ Hall current)

- Efficient confinement only if Hall current does not go to a wall (otherwise Hall effect)

→ Hall current must be closed on itself

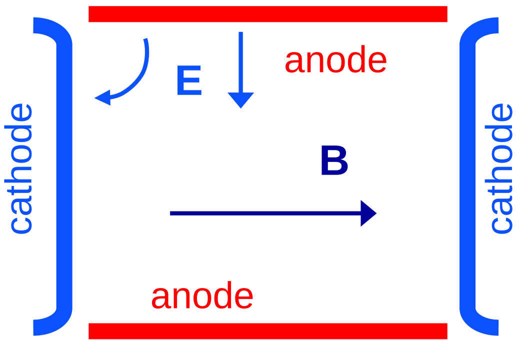
→ Hall current in azimuthal direction of cylindrical device : Closed drift device

→ **Cylindrical configuration with axial E and radial B, or axial B and radial E**

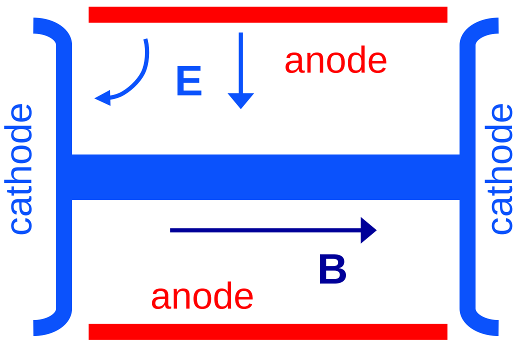
1. Basic physics of low temperature $E \times B$ plasmas

➢ Closed drift devices

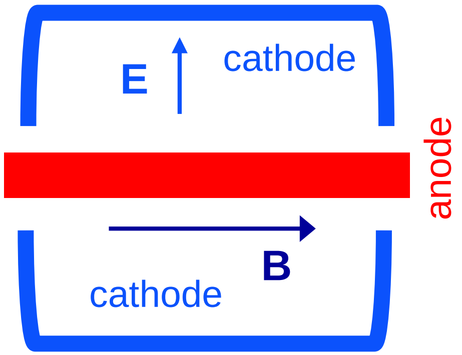
Axial **B**
Radial **E**



(a) Penning cell

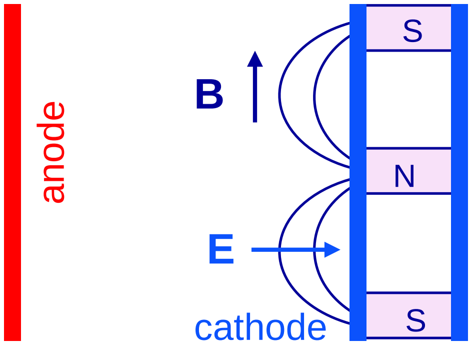


(b) Cylindrical magnetron

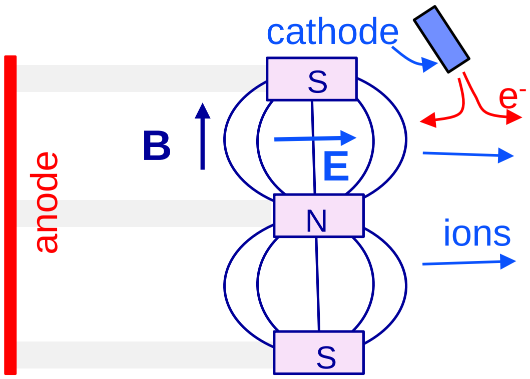


(c) Inverted magnetron

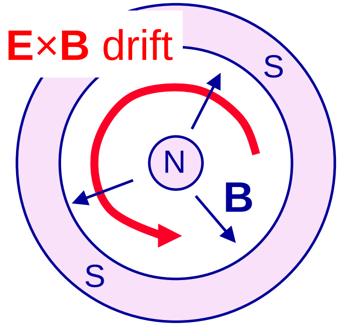
Axial **E**
Radial **B**



(d) Planar magnetron



(e) Hall ion source



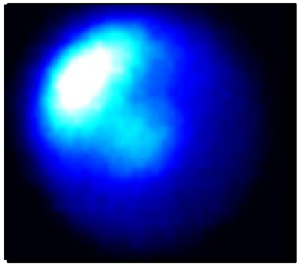
(f) Hall current

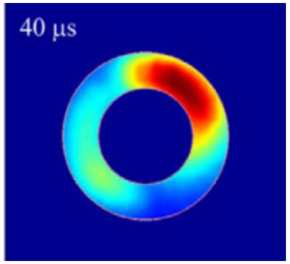
1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Anomalous cross-field transport in closed drift devices

- Confinement of electrons is never as good as predicted by the classical, collisional theory. Instabilities are ubiquitous in $\boldsymbol{E} \times \boldsymbol{B}$ plasmas used in applications → « anomalous » cross-field transport
- Electron mobility // to electric field is reduced but is much larger than predicted by classical, collisional theory
- Unstable fluctuations in a large range of frequencies and wavelengths: kHz to 10's MHz, fraction of mm to cms
- Self-organization into coherent structures (spokes) rotating in the $\boldsymbol{E} \times \boldsymbol{B}$ direction typically occur in Hall thrusters for space propulsion or in magnetron discharges for plasma processing
- In partially magnetized plasmas, instabilities are often due to the large difference between azimuthal drift velocities of electrons and ions → charge separation

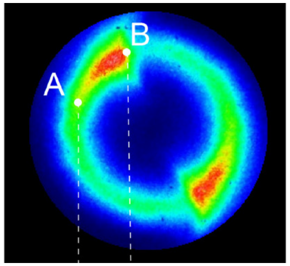
Hall thrusters


Ellisson, Raitses & Fisch, POP (2012)

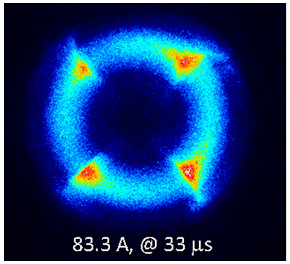


Mazouffre et al. PSST (2019)

magnetron discharges



Hecimovic et al. J phys D (2016)



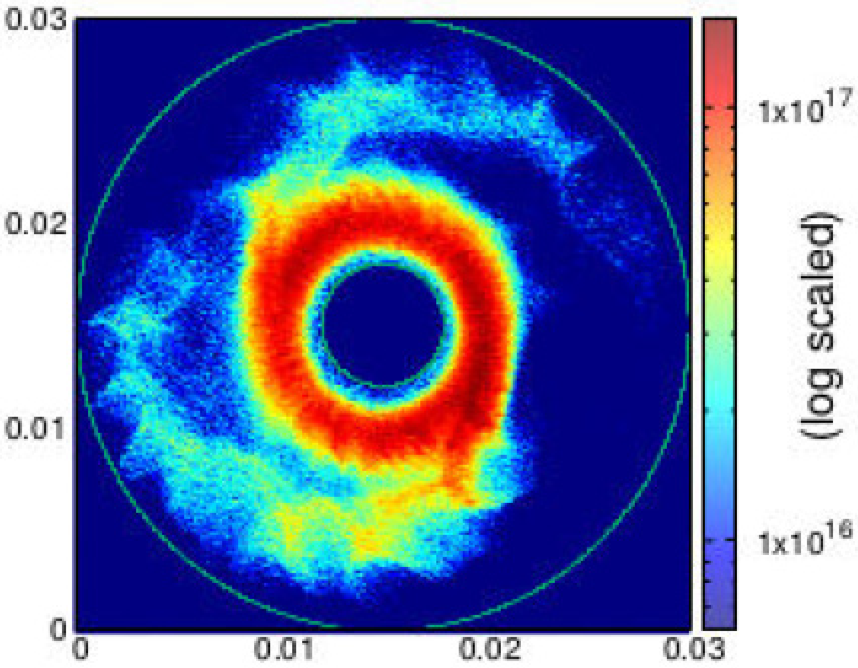
Anders, JAP 2017

1. Basic physics of low temperature $E \times B$ plasmas

➢ Anomalous cross-field transport in closed drift devices

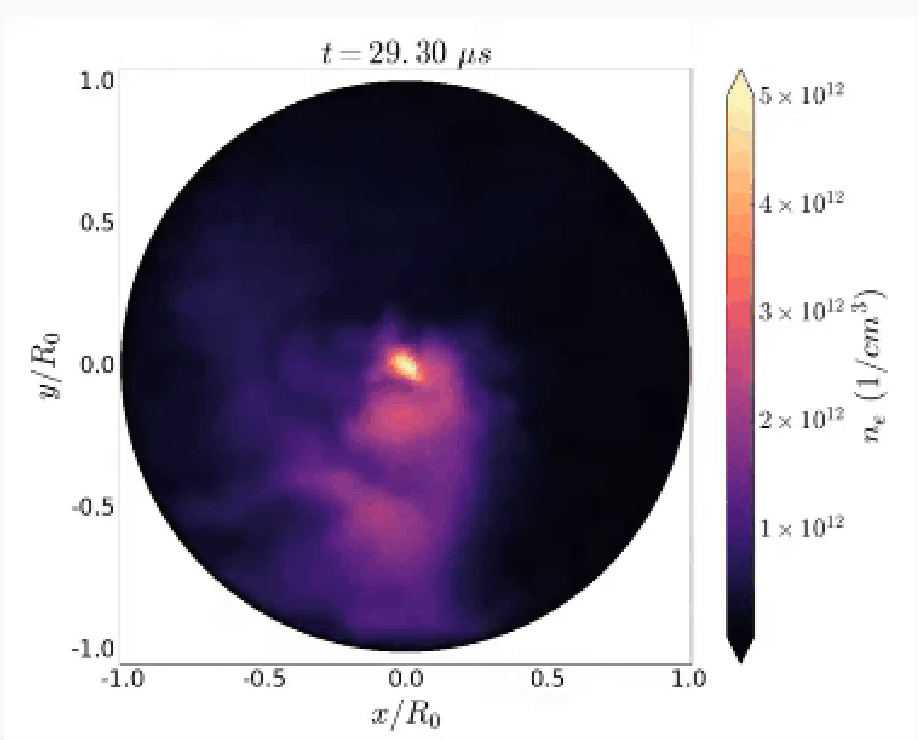
- Rotating non-uniformities are also present in simulations.
- For example, Particle In Cell Monte Carlo Collisions simulations of

cylindrical magnetron discharge



M Sengupta, A Smolyakov, Y Raitses,
J. Appl. Phys. 129, 223302 (2021)

Penning discharge

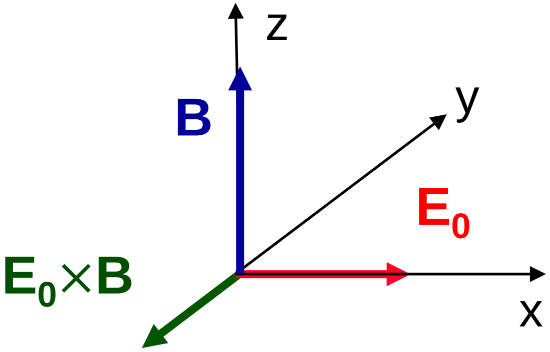


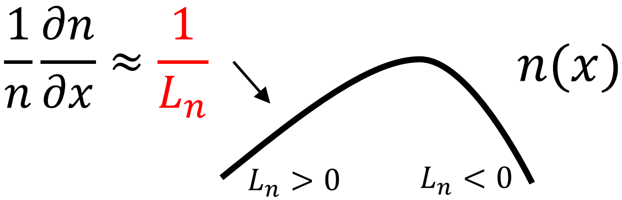
AT Powis, JA Carlsson, ID Kaganovich; Y Raitses, A Smolyakov
Phys. Plasmas 25 072110 (2018)

1. Basic physics of low temperature $E \times B$ plasmas

➢ Instabilities and anomalous transport – Dispersion relations

- Study of instabilities by linearization of fluid or kinetic equations
- Example of a typical problem (e.g. Hall thruster, magnetron discharge)
  - Electric field $\boldsymbol{E_0}$ and plasma density gradient $\frac{1}{n}\frac{\partial n}{\partial x} \approx \frac{1}{L_n}$ in x direction
  - Given external magnetic field $\boldsymbol{B}$ in $\boldsymbol{z}$ direction
  - Study the development of instabilities in **y** direction (**E_0×B**)





Plasma density profile

1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Instabilities and anomalous transport – Dispersion relations

- Example of approximation

- **Ions** – continuity and momentum

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla}.(n_i \boldsymbol{v_i}) = 0$$

$$\frac{\partial \boldsymbol{v_i}}{\partial t} + (\boldsymbol{v_i}.\boldsymbol{\nabla})\boldsymbol{v_i} = -\frac{e}{M}\boldsymbol{\nabla\phi}$$

Collisionless ions
Neglect ion pressure
Non-magnetized ions

- **Electrons** – continuity and momentum

$$\frac{\partial n_e}{\partial t} + \boldsymbol{\nabla}.(n_e \boldsymbol{v_e}) = 0$$

$$n_e \boldsymbol{v_e} = n_e \boldsymbol{v_E} + n_e \boldsymbol{v_{pe}}$$

$$= n_e \frac{-\boldsymbol{\nabla}\phi \times \boldsymbol{B}}{B^2} + \frac{\boldsymbol{\nabla}(n_e T_e) \times \boldsymbol{B}}{B^2}$$

Collisionless electrons
Neglect inertia terms in momentum equation
$\boldsymbol{E} \times \boldsymbol{B}$ + diamagnetic drift
Constant electron temperature

Neglect axial electron and ion velocities

- **Quasineutrality**

$$n_i = n_e$$

Frias et al., Phys. Plasmas 19, 072112 (2012), 20, 052108 (2013)

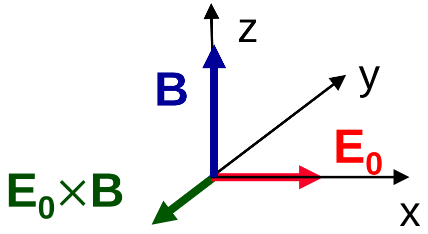
1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Instabilities and anomalous transport – Dispersion relations

- Example of approximation
  - Look for solutions of the form $n = n_0 + \tilde{n}\exp(-i\omega t + \boldsymbol{k}.\boldsymbol{r})$, $\phi = \phi_0 + \tilde{\phi}\exp(-i\omega t + \boldsymbol{k}.\boldsymbol{r})$ with $\tilde{n} << n_0$ and $\tilde{\phi} << \phi_0$, and $\omega = \omega_R + i\gamma$
  - Linearization of the equations above with respect to $\tilde{n}$, $\tilde{\phi}$
  - Growth of the instability if imaginary part of ω is positive: $\gamma > 0$

⇨ Dispersion relation

$$\boxed{\frac{k^2 c_s^2}{\omega^2} = \frac{\omega_*}{\omega - \omega_0}}$$



$\omega_0 = k v_{E0}$ $\qquad v_{E0} = -\frac{E_0}{B}$ $\qquad \boldsymbol{E} \times \boldsymbol{B}$ drift $\qquad c_s^2 = \frac{e}{M} T_e$

$\omega_* = k v_*$ $\qquad v_* = -\frac{T_e}{L_n B}$ $\qquad$ diamagnetic drift

1. Basic physics of low temperature $\boldsymbol{E \times B}$ plasmas

➢ Instabilities and anomalous transport – Dispersion relations

- Example of approximation

$$\boxed{\frac{k^2 c_s^2}{\omega^2} = \frac{\omega_*}{\omega - \omega_0}}$$

Low frequency solutions

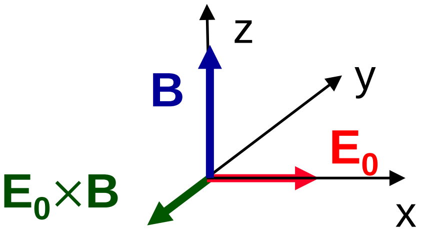
$\omega \ll \omega_0$

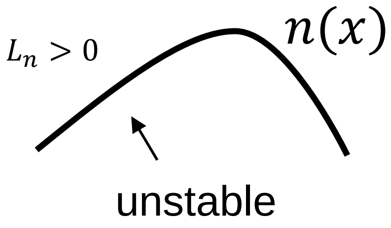
$$\boxed{\frac{k^2 c_s^2}{\omega^2} \approx -\frac{\omega_*}{\omega_0} = -\frac{v_*}{v_{E0}}}$$

Solutions with positive imaginary part → Collisionless **Simon-Hoh instability**

i.e. if electric field and plasma density gradient are in the same direction $\boldsymbol{E . \nabla n > 0}$

very common situations in low temperature magnetized plasmas





1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Instabilities and anomalous transport – Dispersion relations

If electron inertia and gyroviscosity, and e- collisions with neutrals are taken into account, dispersion relation becomes

$$\boxed{\frac{\omega_* + k^2\rho_e^2(\omega - \omega_0 + i\nu_{en})}{\omega - \omega_0 + k^2\rho_e^2(\omega - \omega_0 + i\nu_{en})} = \frac{k^2 c_s^2}{\omega^2}}$$

$\omega_0 = -kv_{E0}$	$\boldsymbol{E} \times \boldsymbol{B}$ drift
$\omega_* = -kv_*$	Diamagnetic drift
c_s	Ion sound speed
ρ_e	Larmor radius
ν_{en}	Electron neutral col freq

→ High frequency, small wavelength, collisional extension of Simon-Hoh instabilities occur due to combined effects of electric field, density gradient and collisions

See A. Smolyakov et al., Plasma Phys. Control. Fusion 59 01404041 (2017)
J.P. Boeuf and A. Smolyakov, Phys. Plasmas 30, 050901 (2023)

1. Basic physics of low temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

➢ Modeling approaches

- Theory can describe only linear development of instabilities. Instabilities does not grow indefinetly. Saturation of instability must be studied with numerical models

- Practically, numerical fluid models are useful but not self-consistent (imposed electron velocity distribution function) + numerical difficulties because of large anisotropy of electron conductivity

→ **Particle-In-Cell Monte Carlo Collisions** (PIC MCC) models to study instabilities

- 3D Particle models still expensive → 2D models

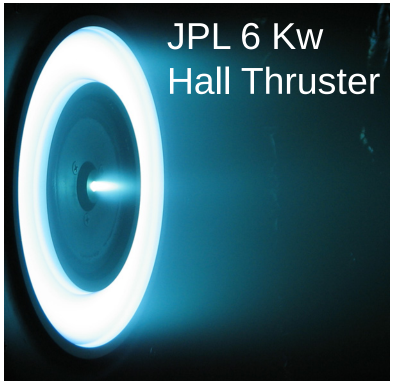
- 2D models must contain azimuthal direction to describe $\boldsymbol{E} \times \boldsymbol{B}$ instabilities

- To understand the physics: simplified, or reduced models to study specific types of instabilities: separate different mechanisms, different time scales etc…

2. Rotating instabilities in $E \times B$ discharge devices

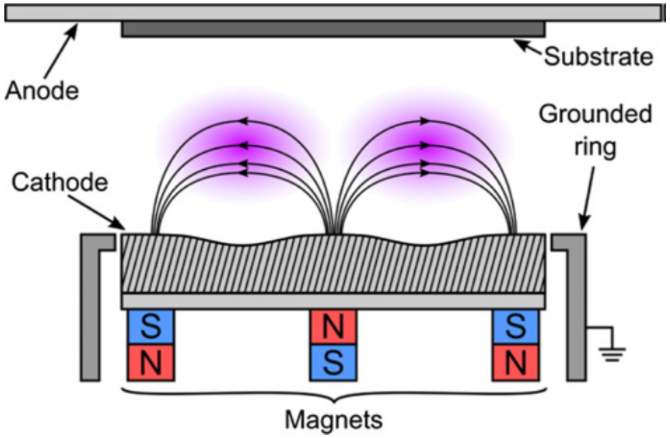
➢ Two examples of rotating instabilities

- Microinstabilities **in Hall thrusters** for spacecraft propulsion



D Goebel & I Katz Fundamental of EP (2002)
JP Boeuf, Tutorial, J.Appl. Phys (2017)

- Macroinstabilities in **Magnetron discharges** for sputtering/deposition

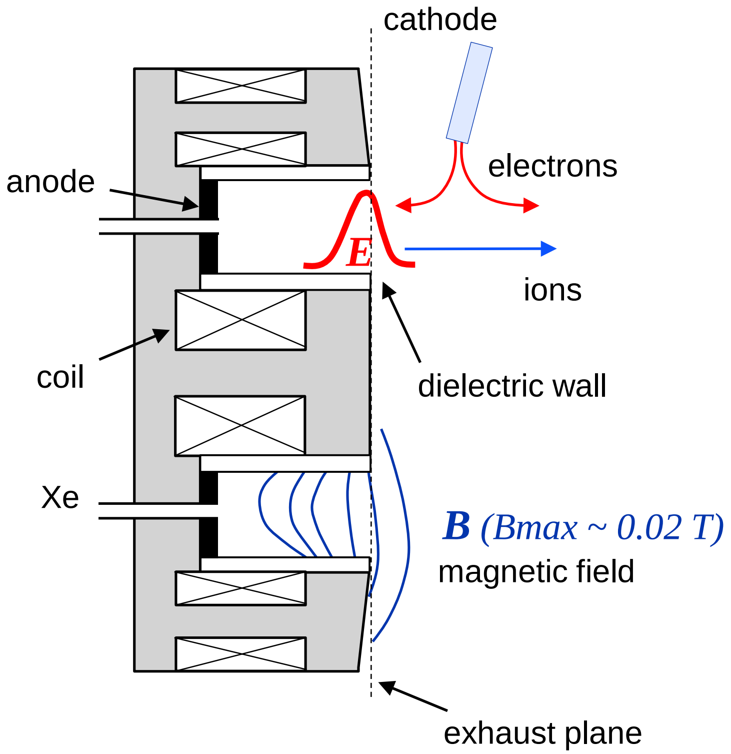


A Anders, J. Appl. Phys. (2017)
JT Gudmundsson, PSST 29 113001 (2020)

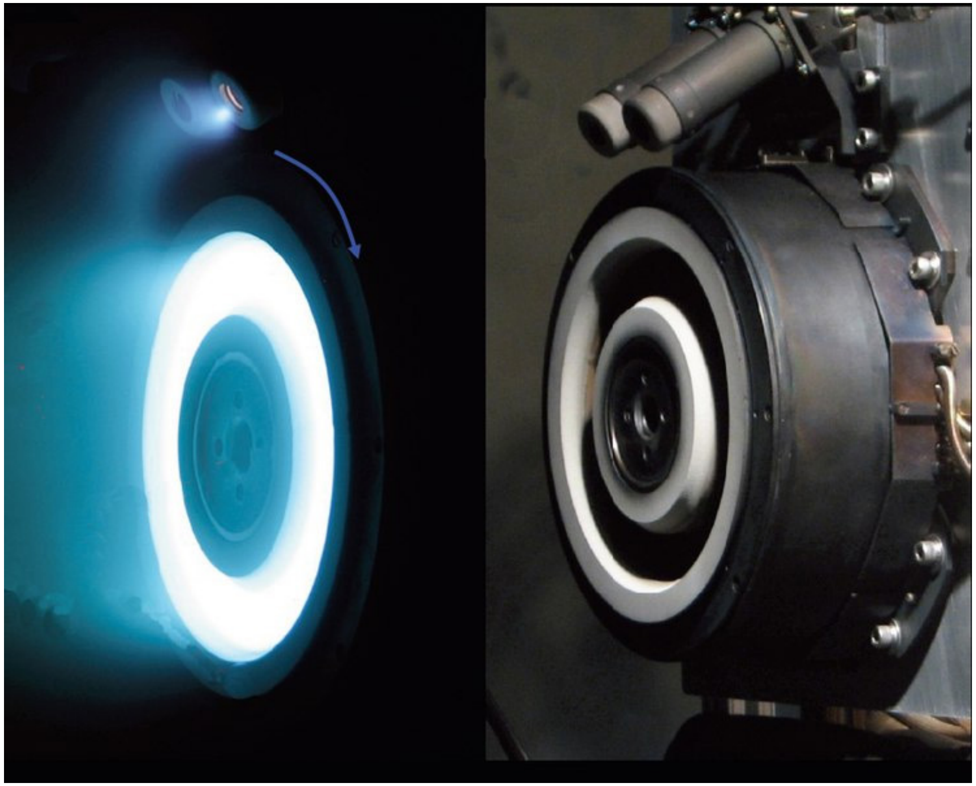
2. Rotating instabilities in $E \times B$ discharge devices

Hall thruster

- Used on many telecom satellites (100 W → 10 kW)
- Work well but physics is not well understod
- No predictive model – Empirical conception
- Instabilities and anomalous electron transport
- Ion extraction without grids → thrust



Electric as large as 400 V/cm generated over 1 cm (>> λ_{De}) in quasineutral high density plasma !



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➢ Hall thruster

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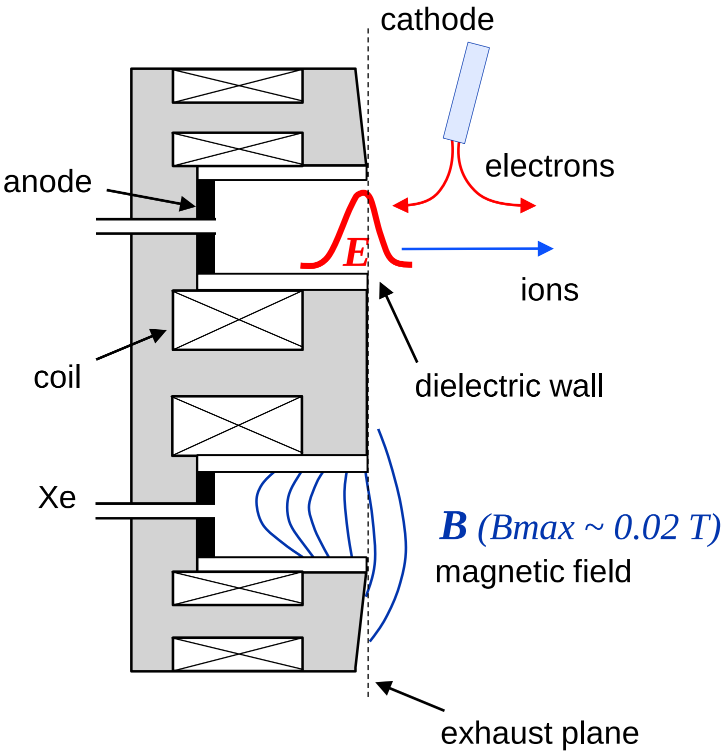
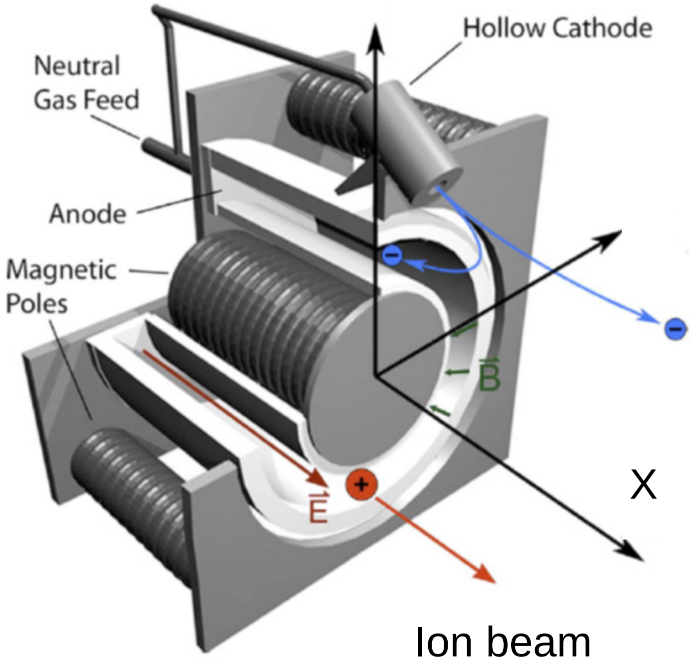


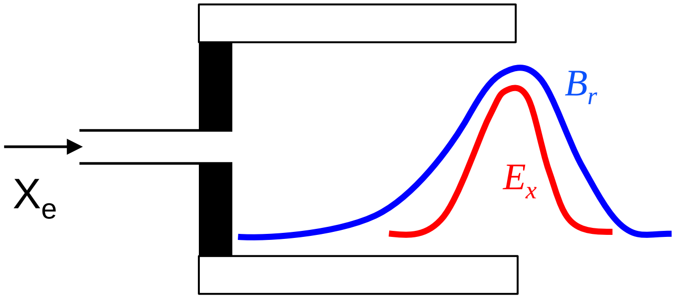


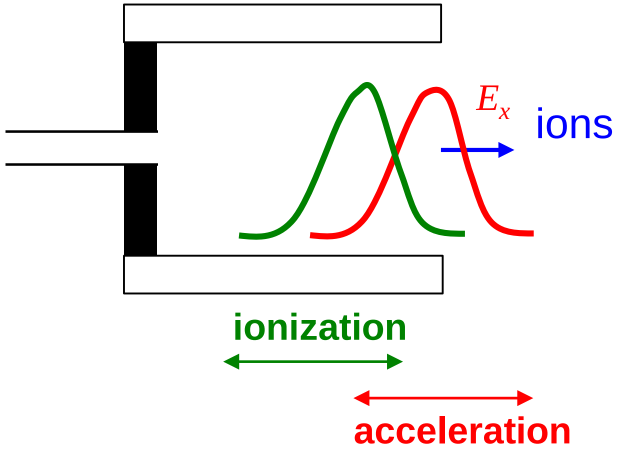
kW thruster: ~ 300 V, 4 A, 5 mg/s Xenon

Electric as large as 400 V/cm generated over 1 cm (>> λ_{De}) in quasineutral high density plasma !

2. Rotating instabilities in $E \times B$ discharge devices

➢ Hall thruster



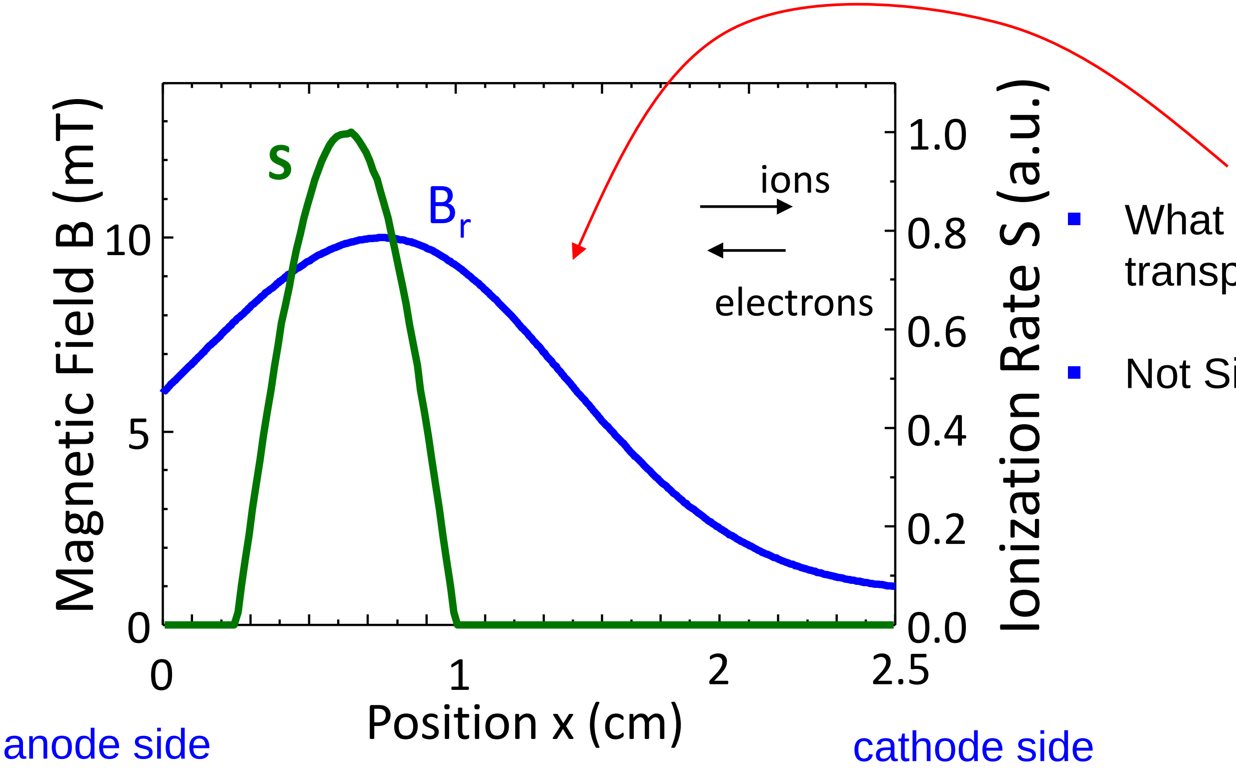


- Small electron conductivity in large radial magnetic field B_r region → large axial electric E_x ($J_{ex} = \sigma E_x$)
- Ionization region upstream of acceleration region → very low neutral density in acceleration region
- Mechanism of cross-field electron transport in ~collisionless region between external cathode and ionization region ?

2. Rotating instabilities in $E \times B$ discharge devices

➢ Simplified 2D PIC MCC axial-azimuthal model of Hall thruster

- (x,z) **collisionless** PIC simulation with given source term (ionization rate) and given electron current injected on the cathode side
- Given magnetic field profile and applied voltage (200 V)
- Goal: study cross-field transport between cathode and ionization region

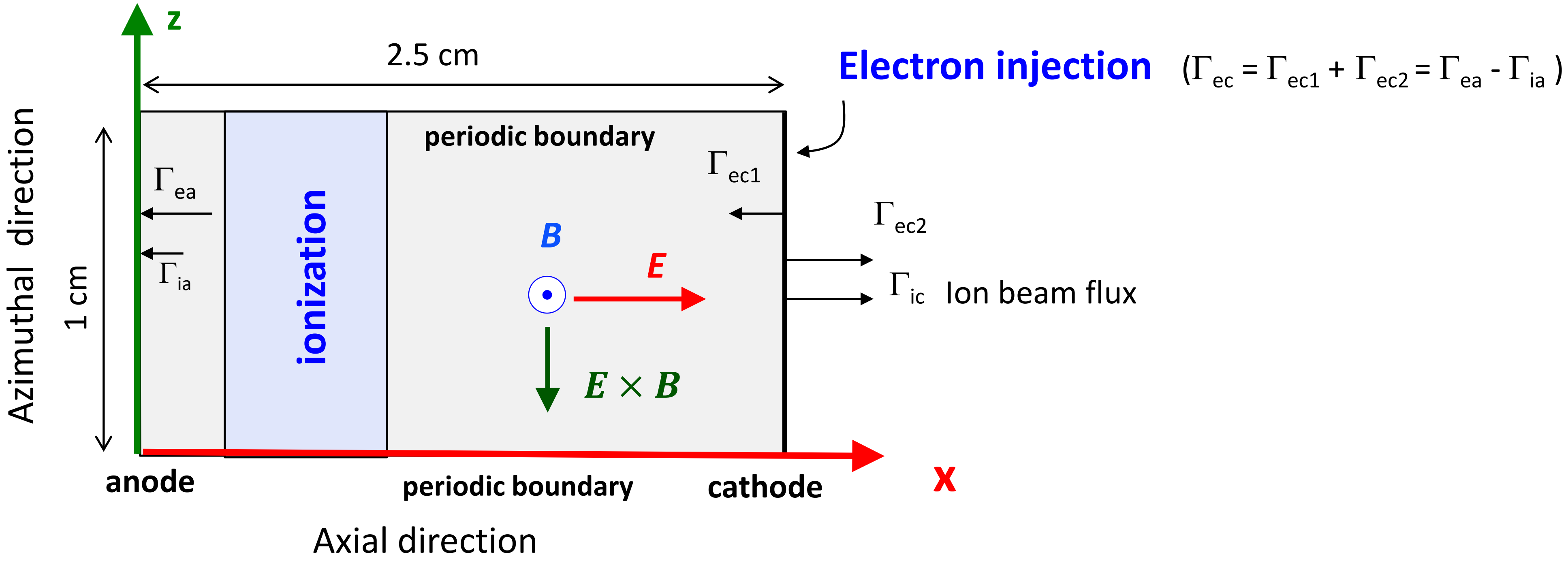


- What type of instability allows electron transport in this ~ collisionless region ?
- Not Simon-Hoh ($\boldsymbol{E}.\boldsymbol{\nabla}\boldsymbol{n} < \boldsymbol{0}$)

2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

2D PIC MCC axial-azimuthal model of Hall thruster

- Rectangular domain with periodic boundary conditions (azimuthal direction)

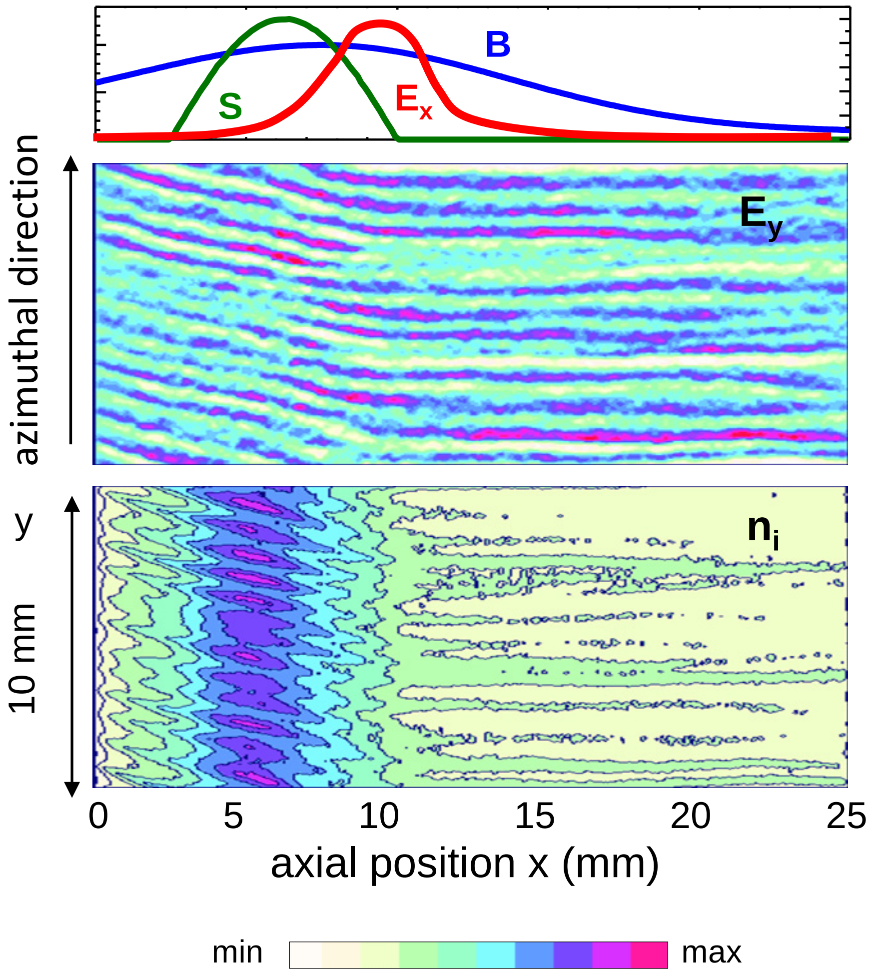


- Typical dimensions (x, y): 2.5 cm x 1 cm

2. Rotating instabilities in $E \times B$ discharge devices

➢ **Example 1**: 2D PIC MCC axial-azimuthal model of Hall thruster

Simulation is consistent with theory of Electron Cyclotron Drift Instability: ECDI



- Collisionless electron transport across B field
- Effective (anomalous) collision frequency: several 10^6 s^{-1}, increases with plasma density
- Effective Hall parameter: $h = \frac{v_{ez}}{v_{ex}}$ =several 100

← Azimuthal field E_y [$-5\text{x}10^4$, $+5\text{x}10^4$] V/m

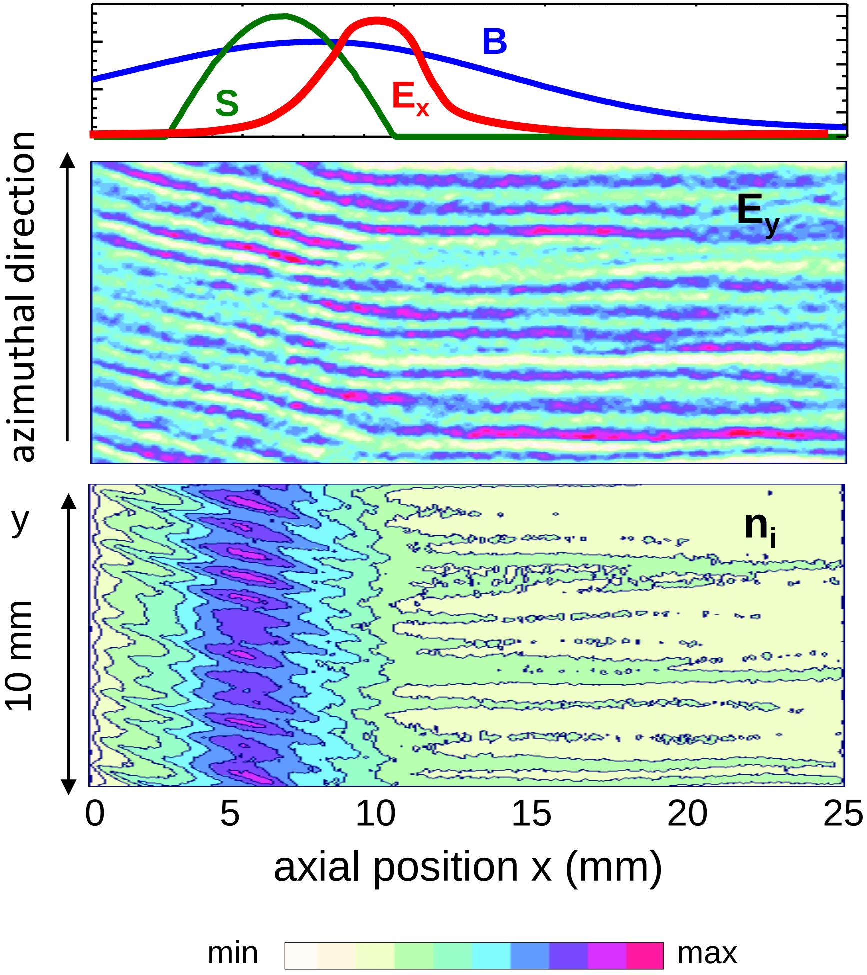
← Positive ion density n_i [$0 – 5\text{x}10^{17}$] m^{-3}

$\boldsymbol{E} \times \boldsymbol{B}$ ↓

2. Rotating instabilities in $E \times B$ discharge devices

➢ Example 1: 2D PIC MCC axial-azimuthal model of Hall thruster

Simulation is consistent with theory of Electron Cyclotron Drift Instability: ECDI



t = 18.85 µs

- Wavelength of the instability ~1 mm
- Wave velocity ~ $5\text{x}10^3$ m/s
- Wave angular frequency ~ $3\text{x}10^7$ rd/s

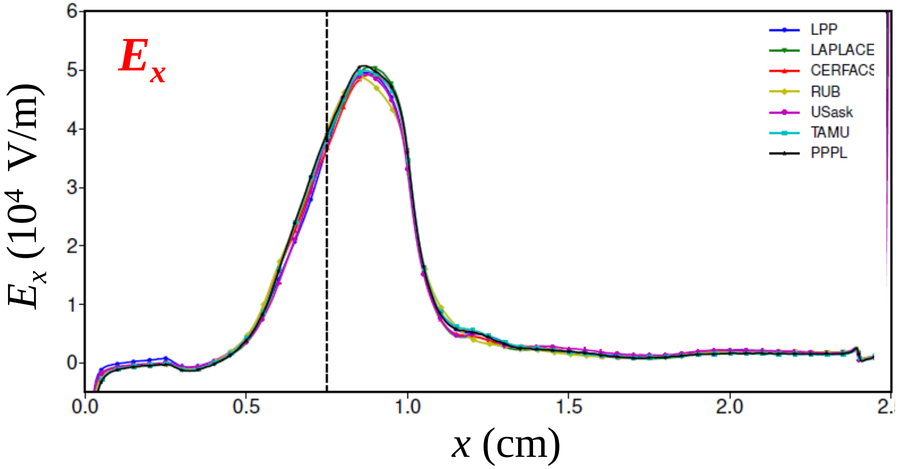
First evidence of ECDI in Hall thrusters in PIC MCC models:
JC Adam, A Heron, G Laval, Phys. Plasmas 11 295 (2004)
Experiments:
Tsikata et al., Phys. Plasmas 16 033506 (2009)

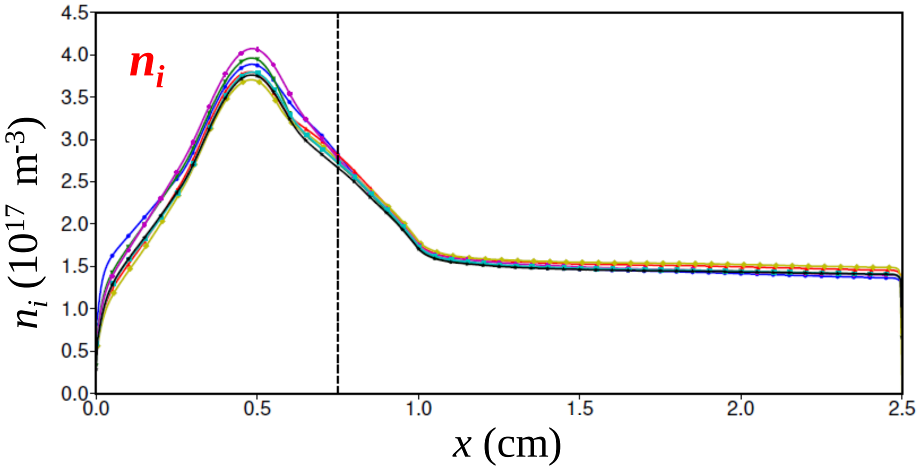
Simplified model
JP Boeuf & L Garrigues, Phys. Plasmas 25 061204 (2018)
T Charoy et al., PSST 28 105010 (2019) (benchmarks)
A Smolyakov et al., Plasma Physics reports, 46 496 (2020)

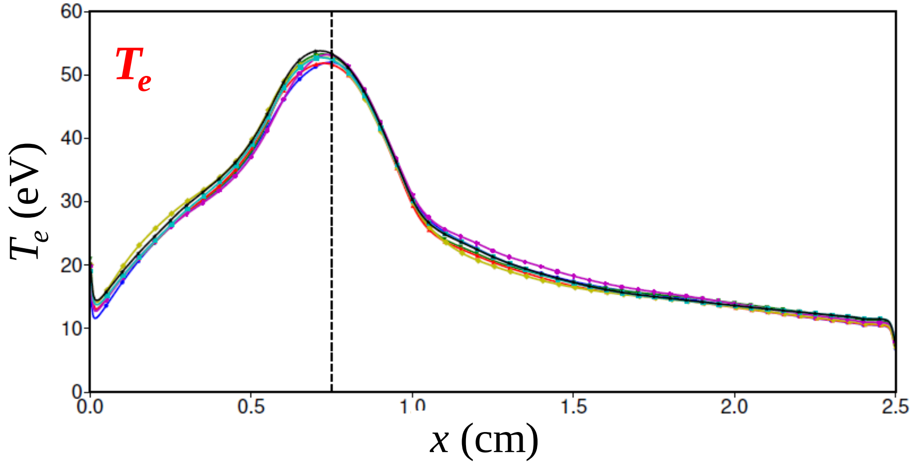
2. Rotating instabilities in $E \times B$ discharge devices

- **Example 1: 2D PIC MCC axial-azimuthal model of Hall thruster**

Time averaged axial profiles







Code Benchmark 7 labs

T Charoy et al., PSST 28 105010 (2019)
W Vilafana et al., PSST 30 075002 (2021)

2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

➢ Theory of ECDI vs PIC simulations vs experiments

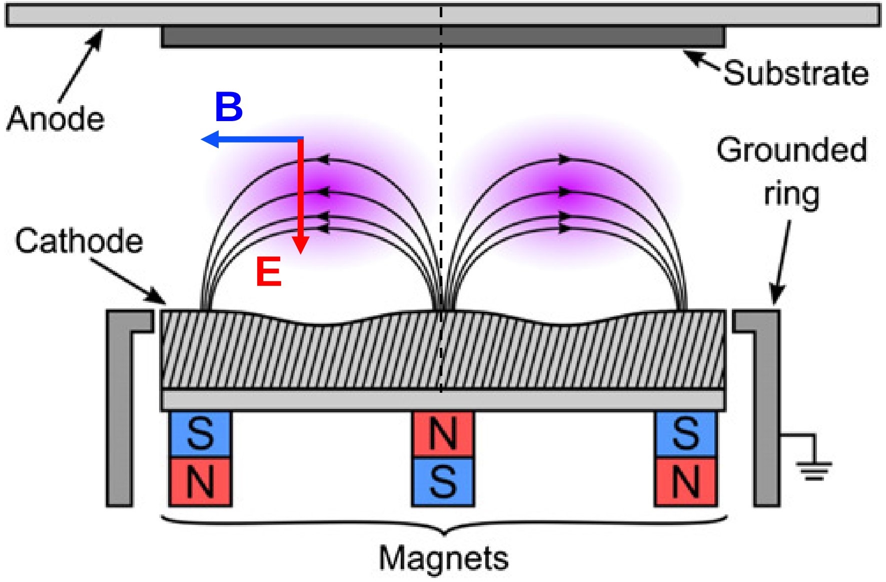
- 2D PIC simulations in good agreement with theory of $\boldsymbol{E} \times \boldsymbol{B}$ Electron Cyclotron Drift Instability.

- Measurements of density fluctuations by collective laser scattering: Tsikata, Gresillon, et al, Phys. Plasmas 16 033506 (2009). Experimental results show instabilities with frequencies and wavelength in agreement with theory and PIC simulations, but amplitude of the fluctuations much smaller than in PIC simulations

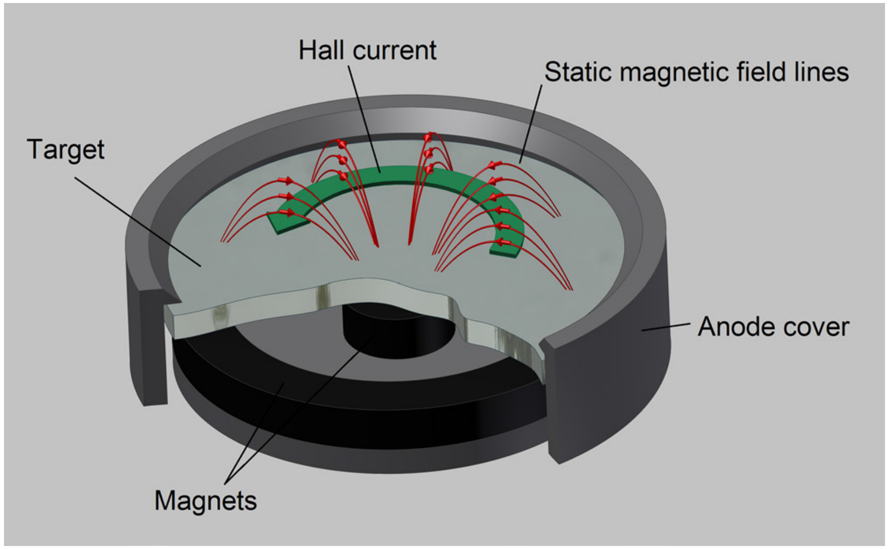
- More work still needed to understand discrepancies between experiment and simulations on the amplitude of fluctuations – Saturation mechanisms ?, 3D effects ?

2. Rotating instabilities in $E \times B$ discharge devices

➢ Magnetron discharges



JT Gudmundsson, PSST 29 113001 (2020)

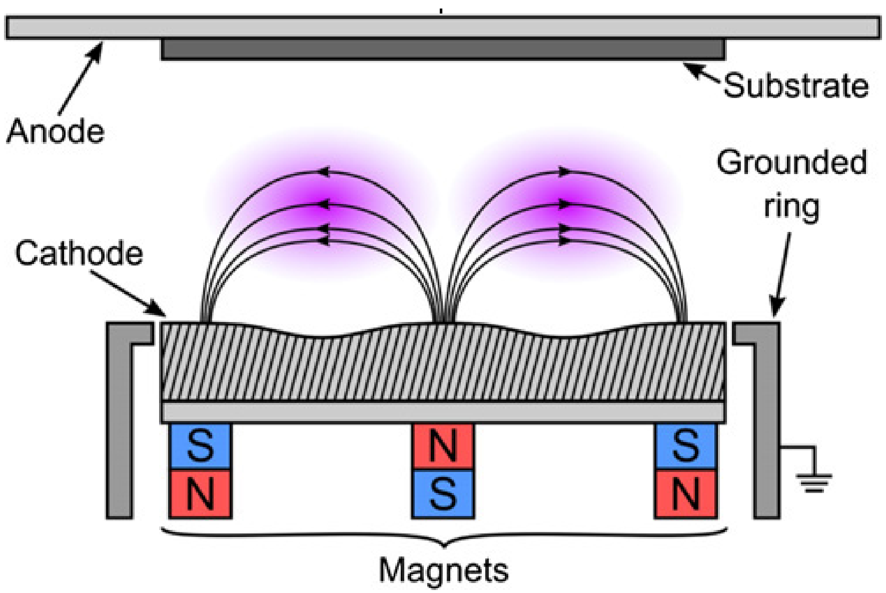


A Hecimovic et al., J Phys D 51 453001 (2018)

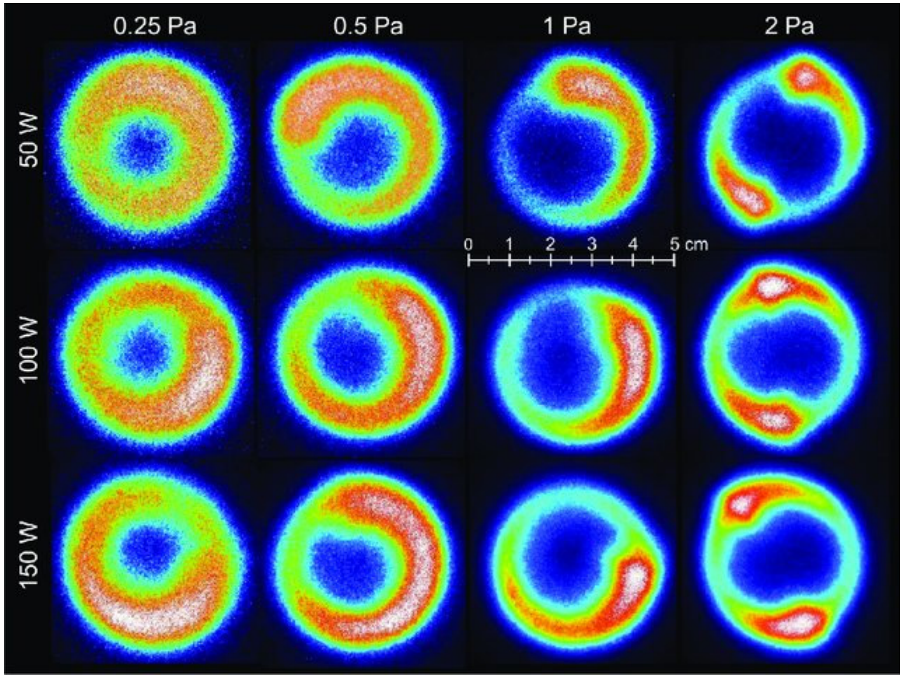
- Cusped magnetic field generated by magnets behind the cathode; azimuthal Hall current
- Operates at pressure ~ 0.1 Pa with dimensions of a few cms
- Ions freely accelerated to the cathode in cathode sheath → sputtering
- Deposition of sputtered metal atoms on substrate
- → application to sputtering and deposition processes.

2. Rotating instabilities in $E \times B$ discharge devices

➢ Magnetron discharges



JT Gudmundsson, PSST 29 113001 (2020)

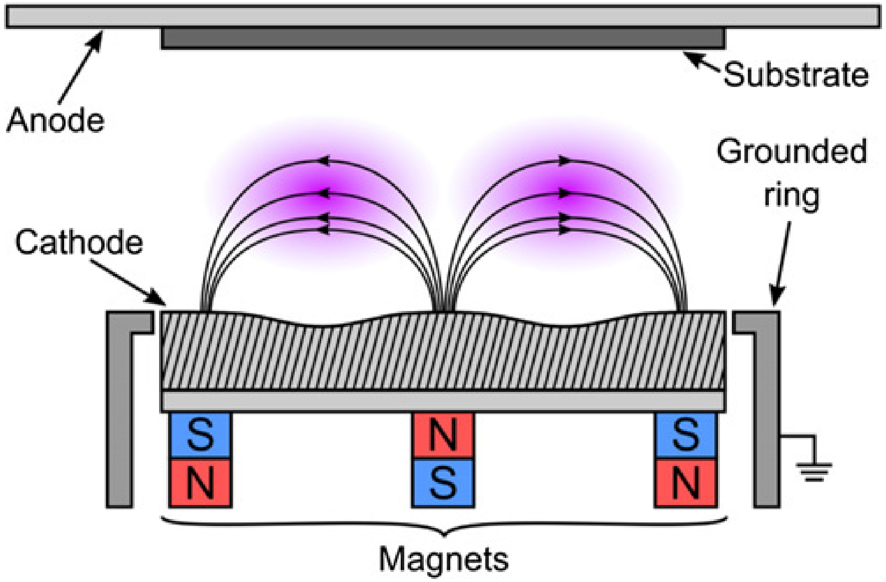


M. Panjan et al., PSST 24 065010 (2015)
M. Panjan, J. Appl. Phys., 125 203303 (2019)

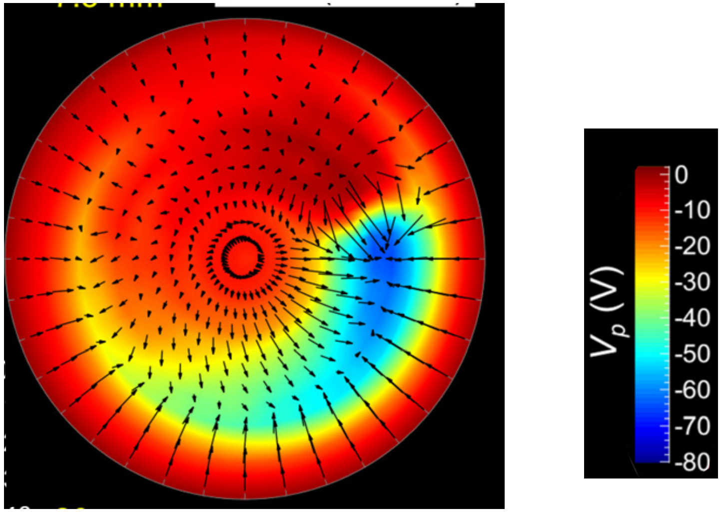
- Discharge is not azimuthally uniform
- Rotating structure (spoke) in azimuthal direction
- Number of modes increases with pressure

2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

Magnetron discharges



JT Gudmundsson, PSST 29 113001 (2020)



M Panjan and A Anders, JAP 121 063302 (2017)
J Held et al. PSST 31 085013 (2022)

- Space and time resolved probe measurements of plasma properties
- Abrupt potential drop at the front of the spoke (several 10s of Volts)
- Attributed to tne formation of a double layer

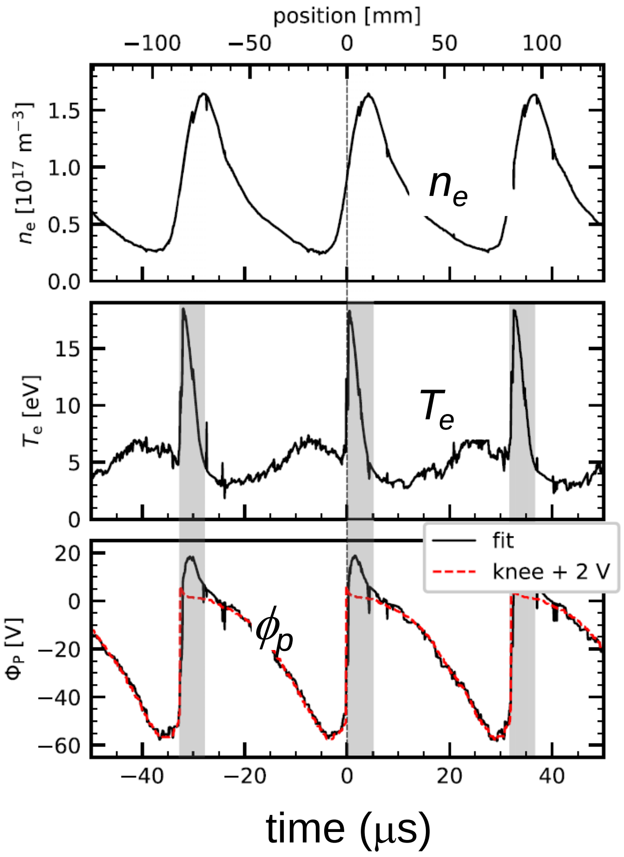
2. Rotating instabilities in $E \times B$ discharge devices

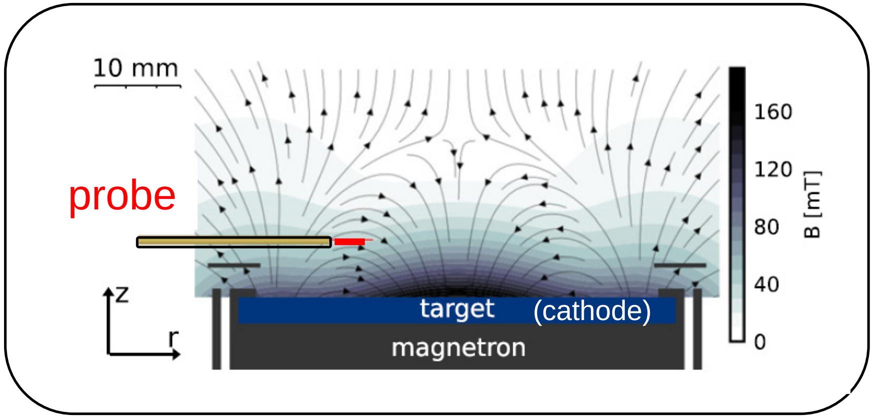
- **Experiments**: **probe measurements of n_e, T_e, ϕ_p, and EEPF at Bochum Univ**

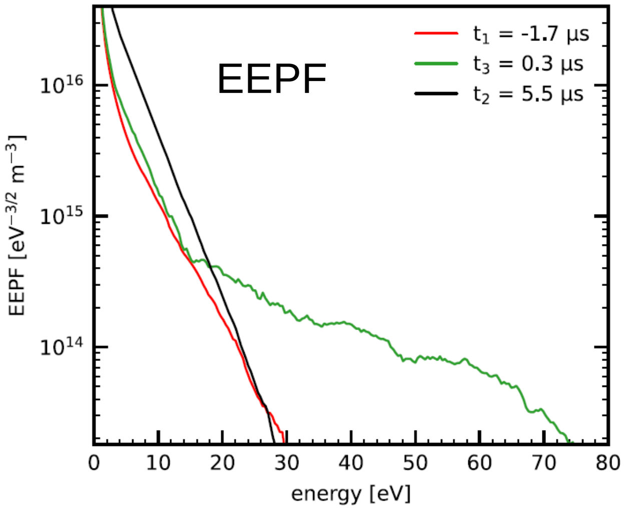
J Held et al., Plasma Sources Sci. Technol. 31 085013 (2022)

Low power, dc magnetron

Argon, 0.6 Pa, B_{max}=120 mT, V=255 V







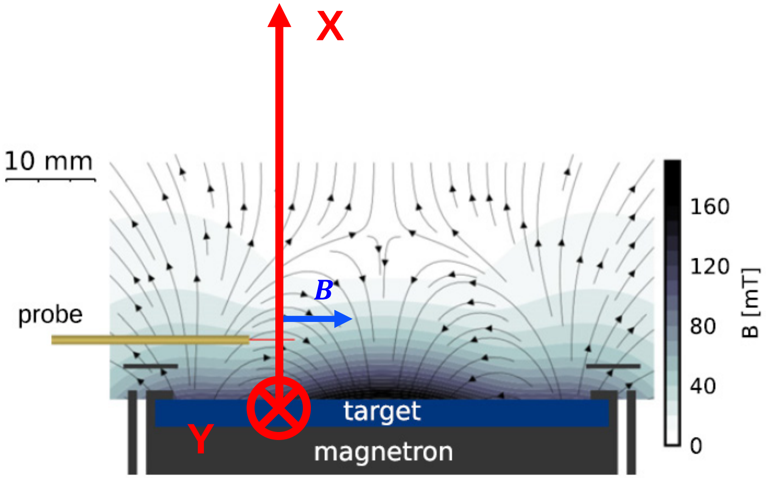
- Observation of one spoke (mode m=1) rotating in the - $E \times B$ direction

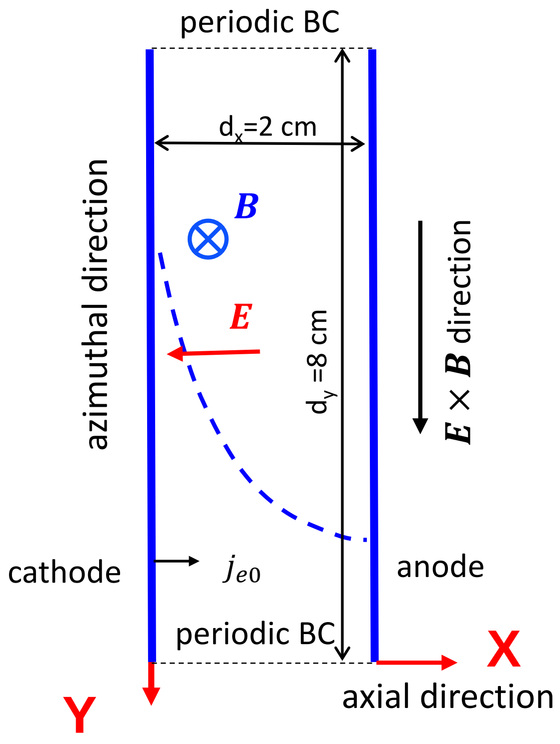
2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

- **PIC MCC simulation of experiment of** J Held et al., PSST 31 085013 (2022)

2D axial-azimuthal PIC MCC Simulation

argon, 0.6 Pa, B_{max}=120 mT, V=260 V





experiment

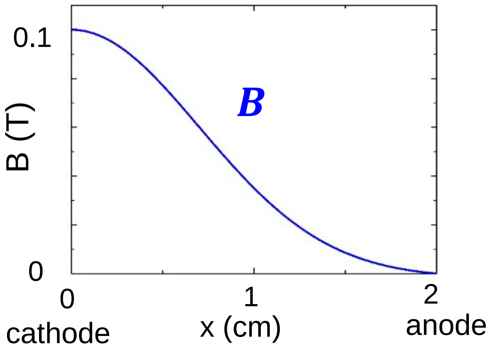
simulation

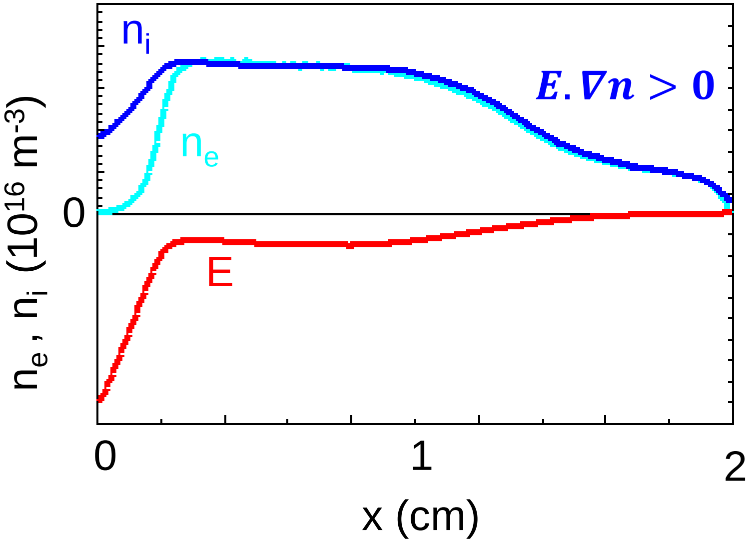
2. Rotating instabilities in $\boldsymbol{E}\times\boldsymbol{B}$ discharge devices

- **1D PIC MCC simulation of experiment of** J Held et al., PSST 31 085013 (2022)

Magnetron discharge,

argon, 0.6 Pa, B_{max}=120 mT, V=260 V



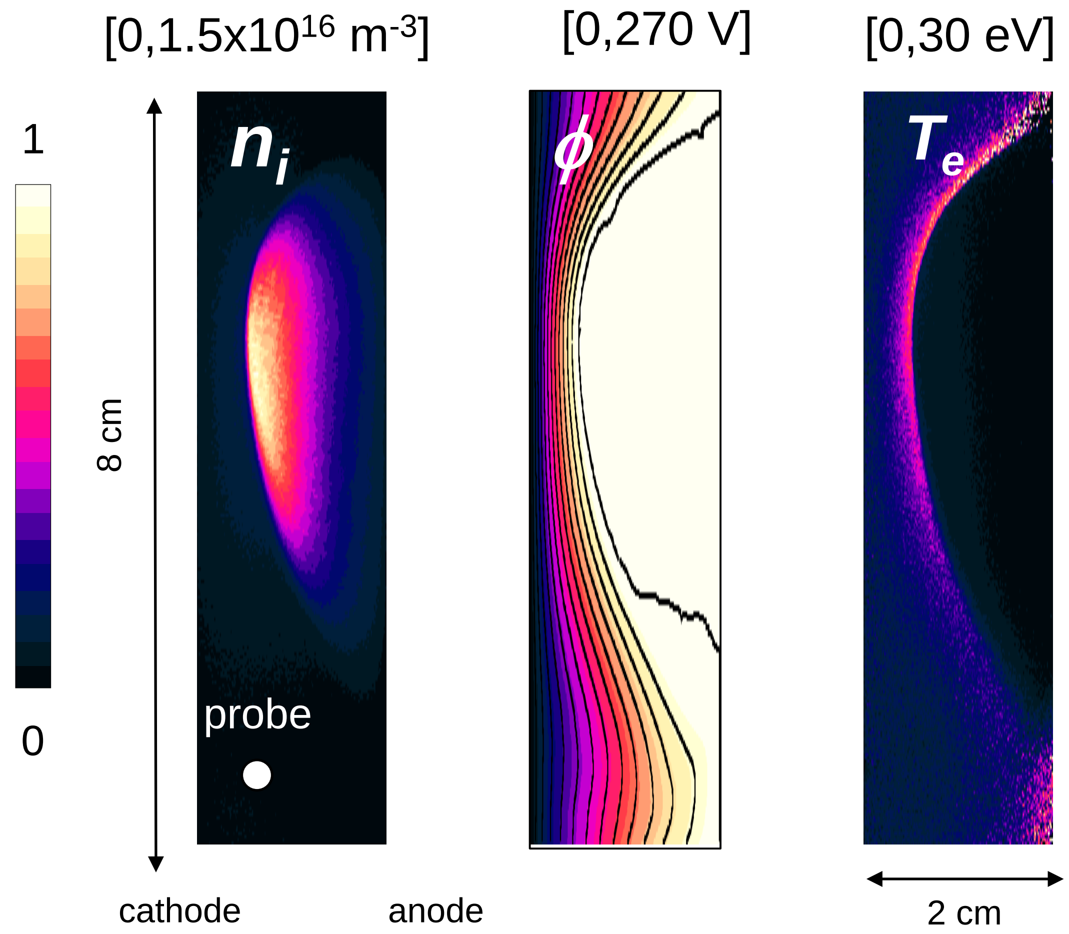


- The 1D solution is stable
- Electric field penetrates in the plasma because of low electron mobility
- A 2D solution including the azimuthal direction is not stable because of Simon-Hoh instability $\boldsymbol{E}.\boldsymbol{\nabla n} > \boldsymbol{0}$ in the plasma (illustrated below)

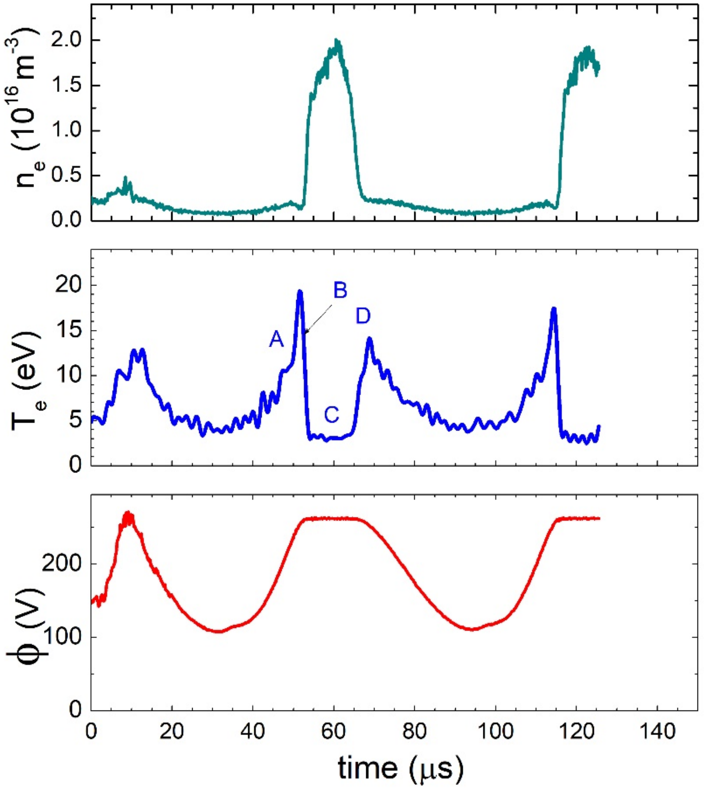
2. Rotating instabilities in $E \times B$ discharge devices

- **2D PIC MCC simulation of experiment of** J Held et al., PSST 31 085013 (2022)

Magnetron discharge, argon, 0.6 Pa, B_{max}=120 mT, V=260 V



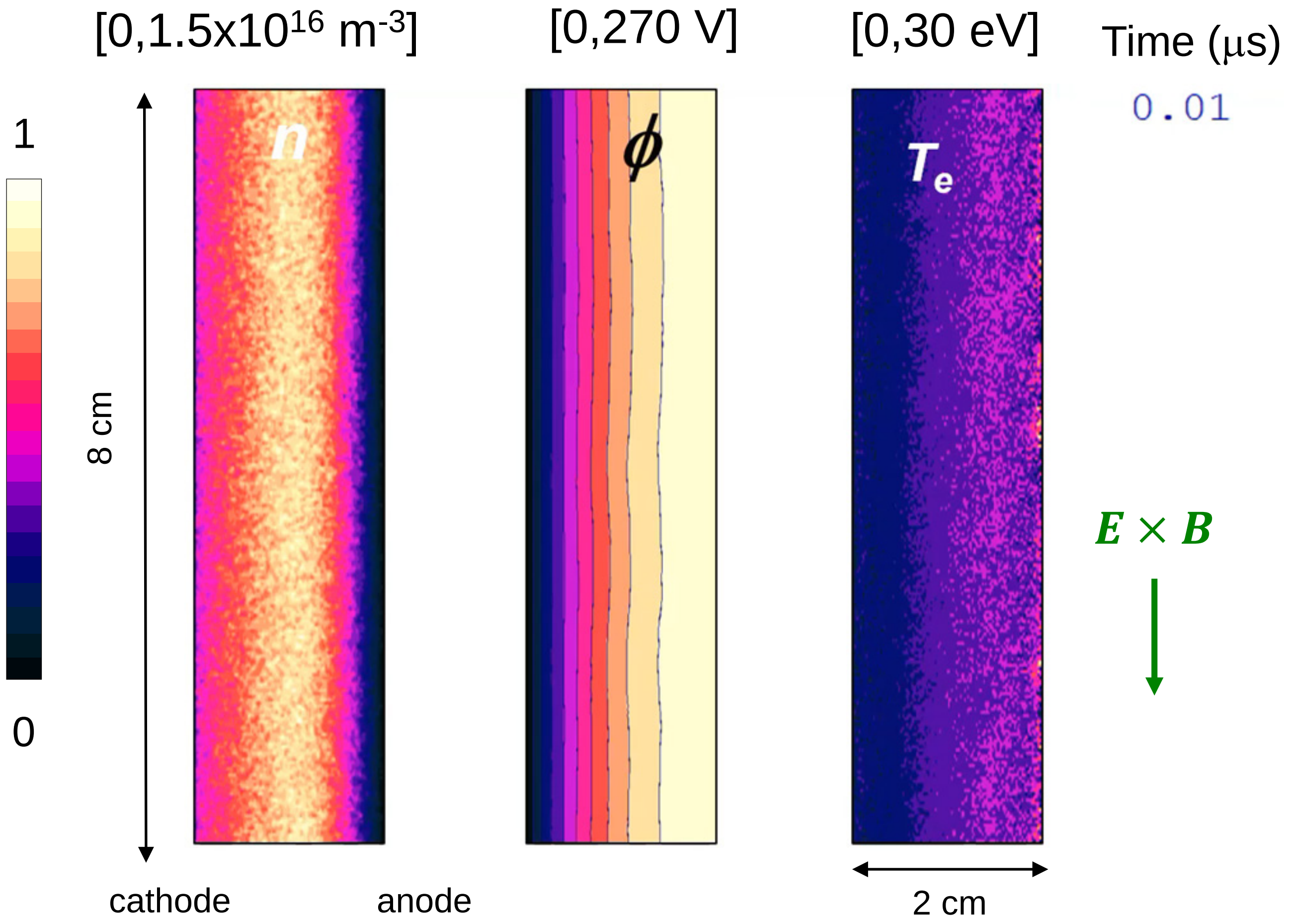
probe measurement (model)



2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

- **2D PIC MCC simulation of experiment of** J Held et al., PSST 31 085013 (2022)

Magnetron discharge, argon, 0.6 Pa, B_{max}=120 mT, V=260 V



Simon-Hoh instability
in linear stage; $\boldsymbol{E}.\boldsymbol{\nabla} n > 0$
(predicted by linear analysis)
+ $\boldsymbol{E} \times \boldsymbol{B}$ motion

ionization wave
in non-linear, saturated stage
- $\boldsymbol{E} \times \boldsymbol{B}$ motion

Smolyakov et al., PPCF 59 014041 (2017)

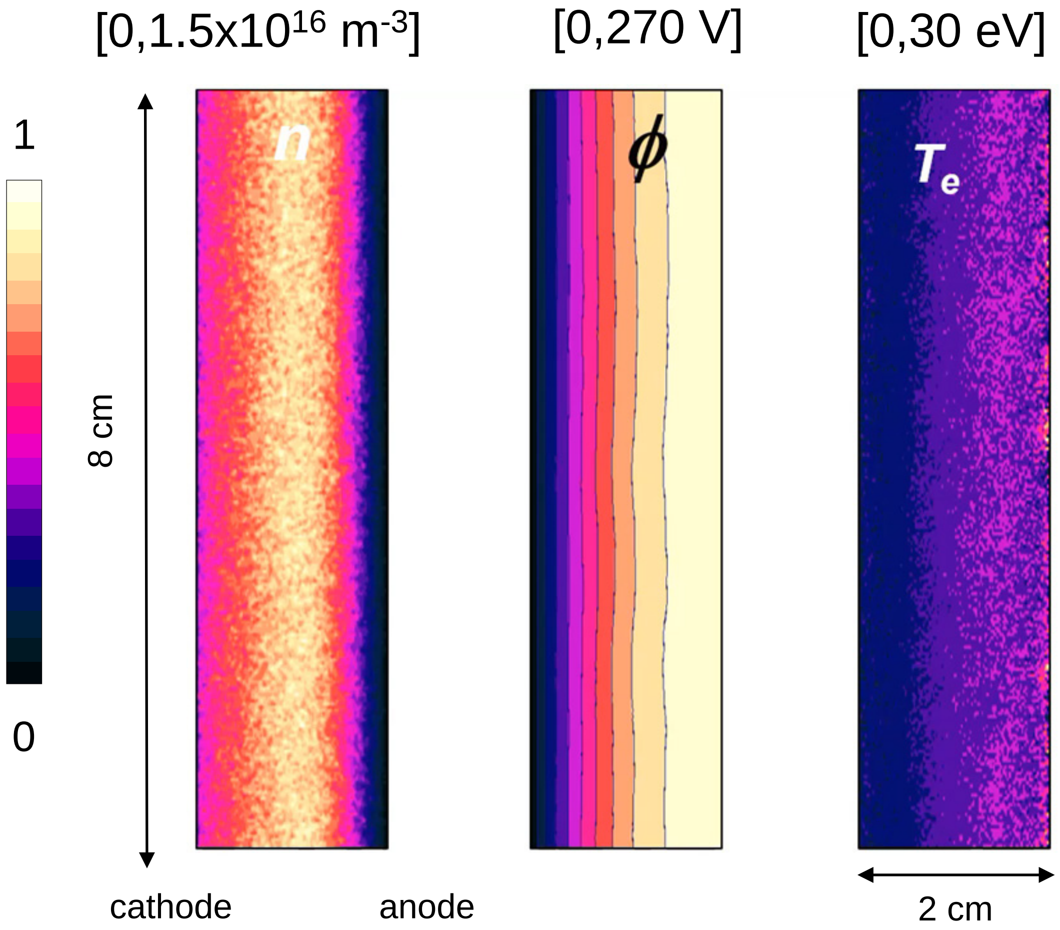
Boeuf, Phys. Plasmas 30 022104 (2023)

Boeuf & Smolyakov,
Phys. Plasmas 30 050901 (2023)

2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

- **2D PIC MCC simulation of experiment of** J Held et al., PSST 31 085013 (2022)

Magnetron discharge, argon, 0.6 Pa, B_{max}=120 mT, V=260 V



Time (µs)

0.01

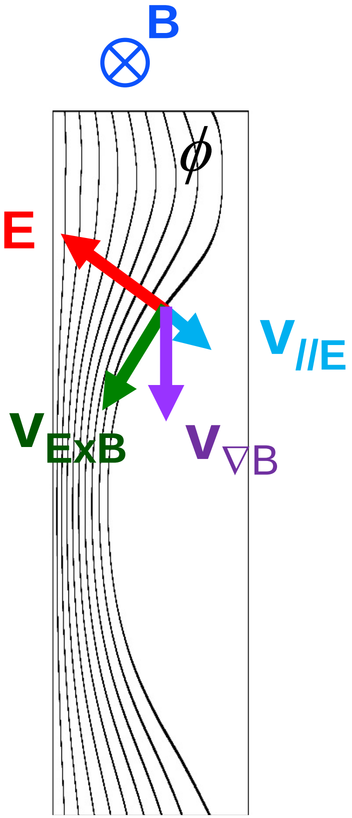
- Large increase of electron temperature in the spoke's front
- Why ?

$\boldsymbol{E} \times \boldsymbol{B}$

2. Rotating instabilities in $E \times B$ discharge devices

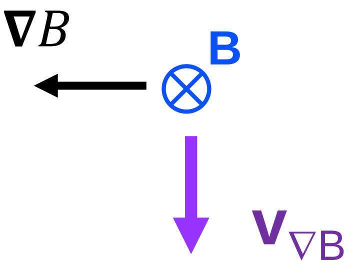
➢ PIC-MCC simulation of magnetron discharge

Mechanisms of electron heating $\boldsymbol{J_e}.\boldsymbol{E} = -en_e\boldsymbol{v}.\boldsymbol{E}$



- Electron heating per electron (eV/s) $\theta = -\boldsymbol{v}.\boldsymbol{E}$

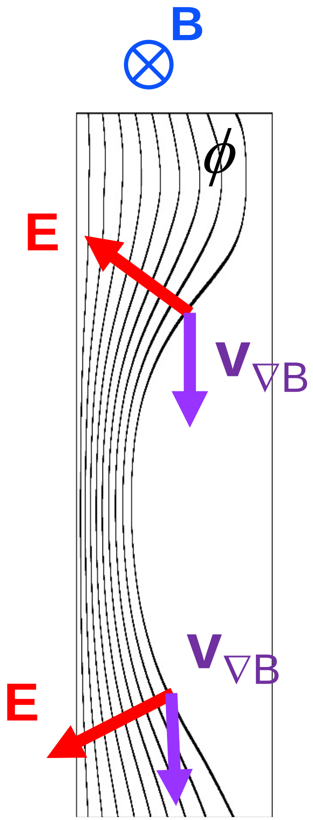
- Electron mean velocity is composed of:
  - **ExB drift velocity** $\boldsymbol{v_{E\times B}} \approx E/B$ $\boldsymbol{v}_{E\times B} \perp \boldsymbol{E} \Rightarrow$ no electron heating
  - **Mean velocity //E** $\boldsymbol{v_{\parallel E}} \approx E/B\ \nu/\omega_{ce}$ small because ν/ω_{ce}<<1
  - **Grad B drift velocity** $\boldsymbol{v_{\nabla B}} = 1/2\ \rho_e v_\perp \boldsymbol{B} \times \boldsymbol{\nabla} B/B^2$



2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

- PIC-MCC simulation of magnetron discharge

Mechanisms of electron heating



- Electron heating per electron (eV/s) $\theta = -\boldsymbol{v}.\boldsymbol{E}$

Grad B drift velocity $\boldsymbol{v}_{\boldsymbol{\nabla B}} = 1/2\ \rho_e v_\perp \boldsymbol{B} \times \boldsymbol{\nabla} B/B^2$

$\theta = -\boldsymbol{v}_{\boldsymbol{\nabla B}}.\boldsymbol{E} > 0$ electron heating

$\theta = -\boldsymbol{v}_{\boldsymbol{\nabla B}}.\boldsymbol{E} < 0$ electron cooling

3. CONCLUSION (1)

- $\boldsymbol{E} \times \boldsymbol{B}$ configurations are used to confine electrons and generate plasmas at low pressures e.g. in ion sources, ion thrusters for space propulsion, magnetron discharges for sputtering-deposition, …
- The physics of $\boldsymbol{E} \times \boldsymbol{B}$ discharges is rich and complex, and not fully understood. Specific properties of instabilities due to partial magnetization
- Important recent progress in the understanding of $\boldsymbol{E} \times \boldsymbol{B}$ plasmas. Synergy between different approaches: theory, experiments, fluid models, kinetic models.
- Work is still needed to develop models with predicting capabilities

3. CONCLUSION (2)

- **LTP community is organizing itself to identify critical questions related to partially magnetized $\boldsymbol{E} \times \boldsymbol{B}$ plasmas, and suggest directions for future research**
  - Review papers

→ **I Kaganovich et al., *Physics of $\boldsymbol{E} \times \boldsymbol{B}$ discharges relevant to plasma propulsion and similar technologies*, Phys.Plasmas 27, 120601 (2020)**

→ **J.P. Boeuf and A. Smolyakov., *Physics and instabilities of low-temperature $\boldsymbol{E} \times \boldsymbol{B}$ plasmas for spacecraft propulsion and other applications,* Phys. Plasmas 30, 050901 (2023)**

- Organization of workshops on $\boldsymbol{E} \times \boldsymbol{B}$ plasmas

  Toulouse 2017, Princeton 2018, Madrid 2022, …

- Organization of benchmarks of codes (and experiments ?)

  https://www.landmark-plasma.com

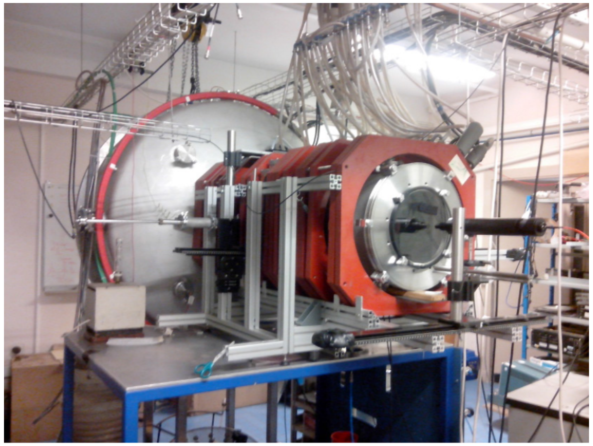
- Copy of similar presentation

  (64th annual meeting of the APS Division of Plasma Physics, 2022)

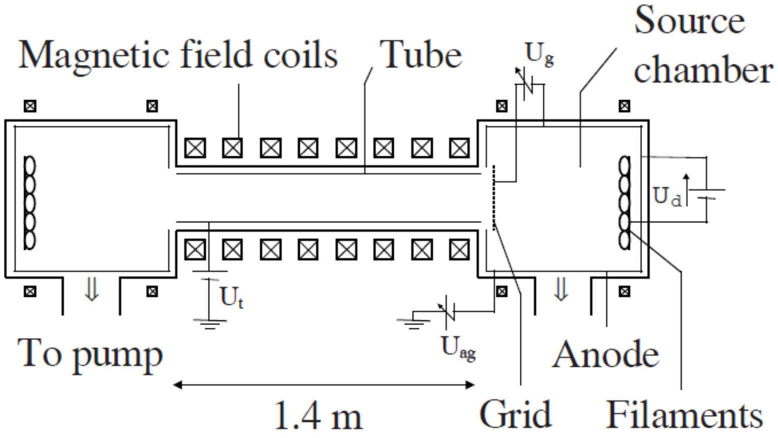
  https://www.jpboeuf.fr/research/conference-presentations

1. Basic physics of low temperature $E \times B$ plasmas

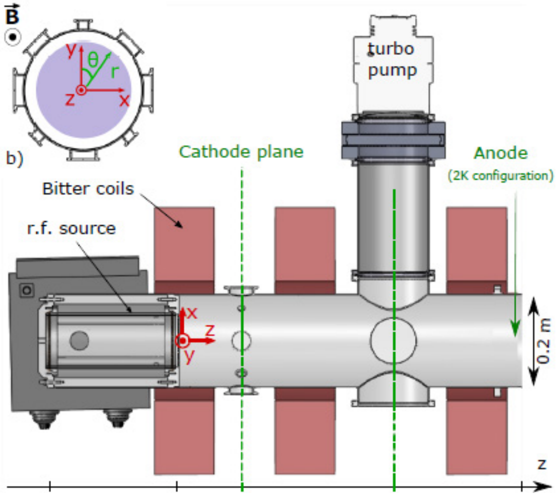
➢ Anomalous cross-field transport in plasma columns



MISTRAL – Marseille



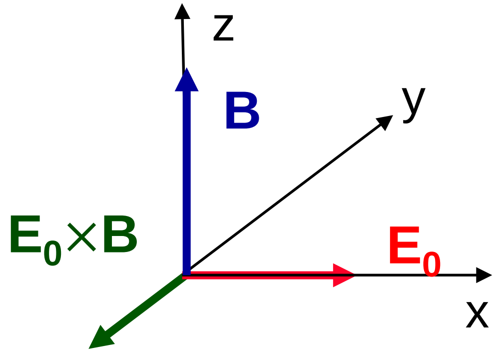
MIRABELLE – Nancy

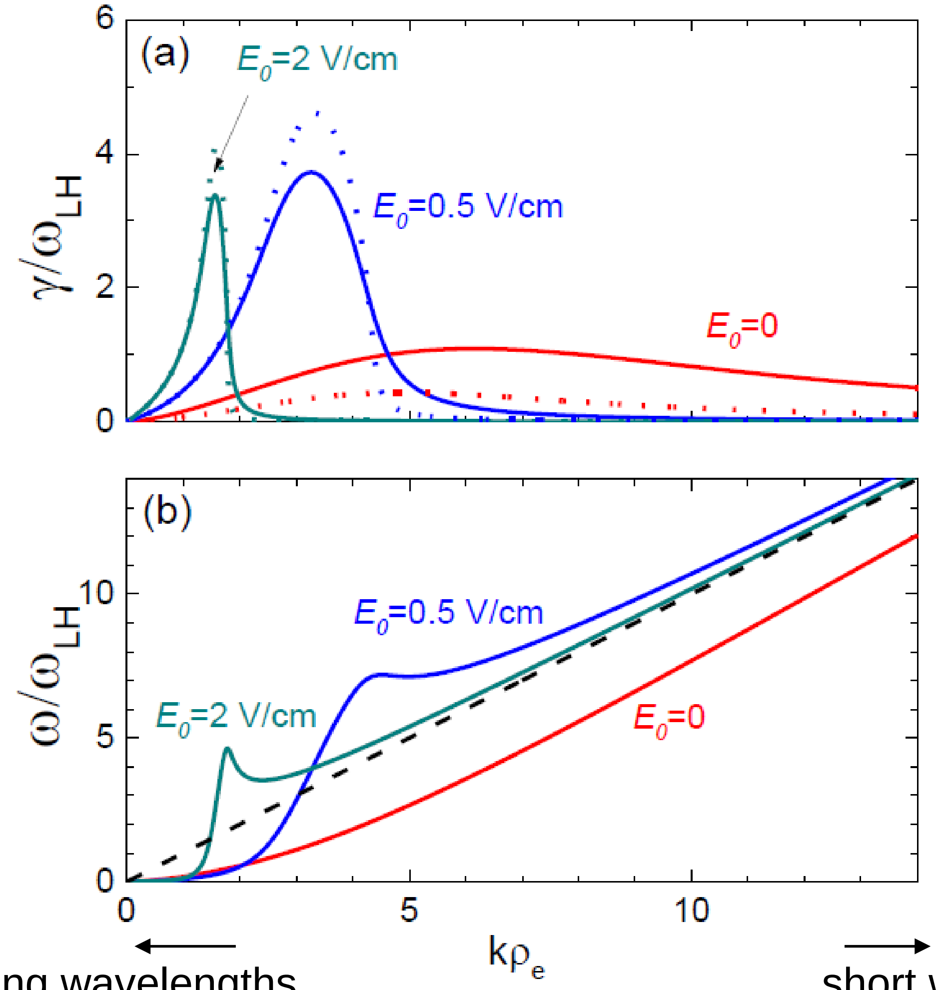


1. Basic physics of low temperature $E \times B$ plasmas

Example of solutions of dispersion relation

$\omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}} = eB/\sqrt{mM}$ (lower hybrid frequency)





long wavelengths

short wavelengths

Calculated for xenon, B=40 Gauss
($\omega_{LH} = 1.4 \times 10^6$ rd/s)
$\nu_{en} = 5 \times 10^6\ \mathrm{s}^{-1}$
$T_e = 5$ eV, $L_n = 1$ cm

Instability wavelength: $\lambda = \frac{2\pi}{k}$

See A. Smolyakov et al., PPCF 59 01404041 (2017)
J.P. Boeuf, Phys. Plasmas 26 072113 (2019)

2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

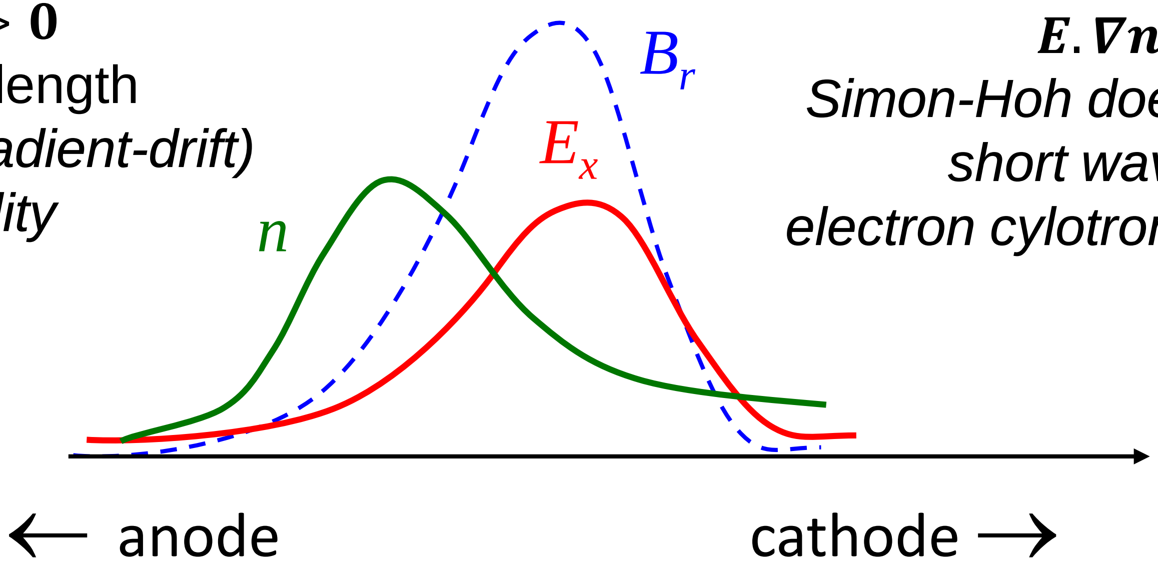
- **Examples: 2D PIC MCC axial-azimuthal model of $\boldsymbol{E} \times \boldsymbol{B}$ discharges**

$\boldsymbol{E}.\boldsymbol{\nabla n} > \mathbf{0}$

long wavelength

Simon-Hoh (gradient-drift)

instability



$\boldsymbol{E}.\boldsymbol{\nabla n} < \mathbf{0}$

Simon-Hoh does not develop

short wavelength

electron cylotron drift instability

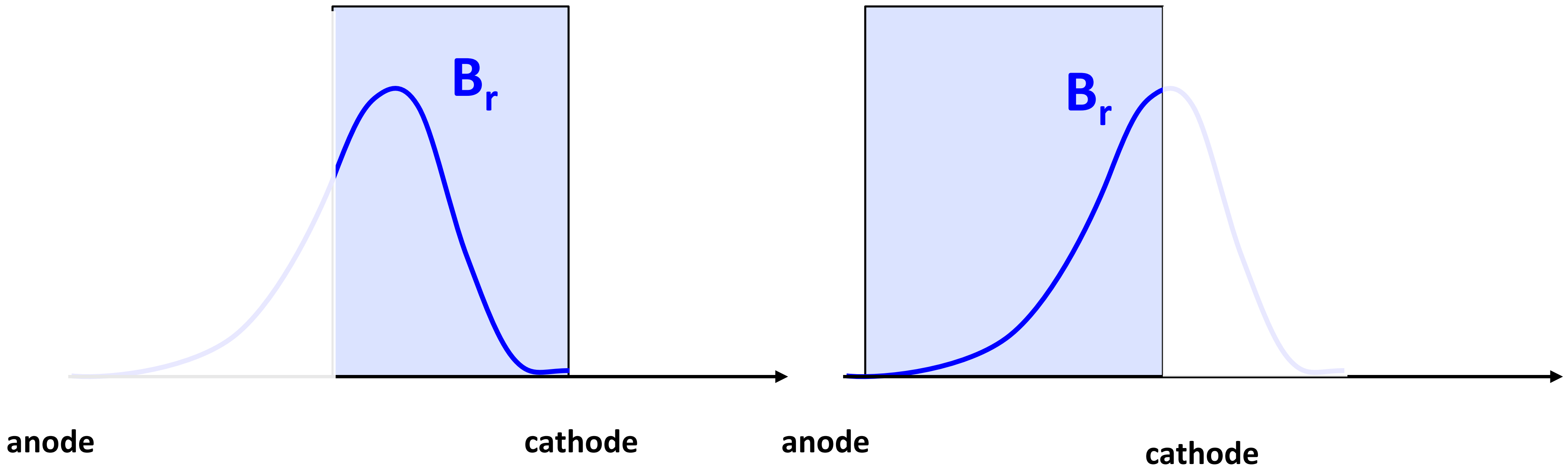
- 2 very different types of instabilities on anode and cathode sides of B_{max}
- On anode side: electric field and density gradient in the same direction
  → Large wavelength, Simon-Hoh instability leading to «Rotating Spoke »
- On cathode side:
  → kinetic micro-instability (Electron Cyclotron Drift Instability)

2. Rotating instabilities in $E \times B$ discharge devices

- **Examples: 2D PIC MCC axial-azimuthal model of $E \times B$ discharges**

Example 1

Example 2



- Simplied model of exhaust region of Hall thruster
- Collisionless
- Given ionization profile

- Simplied model of magnetron or inside channel of Hall thruster
- Collisional
- Self-consistent ionization

2. Rotating instabilities in $\boldsymbol{E}\times\boldsymbol{B}$ discharge devices

➢ Theory of $\boldsymbol{E}\times\boldsymbol{B}$ Electron Cyclotron Drift Instability

- Azimuthal instability found in PIC simulations of Hall thrusters similar to **Electron Cyclotron Drift Instability** studied in the 1970’s in the context of collisionless shocks in space plasmas.

- Instability is triggered by the difference in azimuthal velocities of electrons and ions

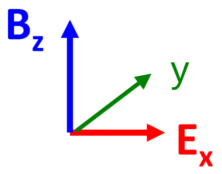
- **Kinetic instability due to coupling between electron Bernstein waves and ion acoustic waves**

- Dispersion equation of electrostatic waves in a hot magnetized electron beam drifting across a magnetic field, with non magnetized ions

$$1+k^2\lambda_{De}^2+g\left(\frac{\omega-k_yV_d}{\Omega_{ce}},\left(k_x^2+k_y^2\right)\rho^2,k_z^2\rho^2\right)-\frac{k^2\lambda_{De}^2\omega_{pi}^2}{\left(\omega-k_xV_{i,b}\right)^2}=0$$

$g(\Omega,X,Y)$ Gordeev function

$$\rho=\frac{\mathrm{v}_{the}}{\Omega_{ce}}$$



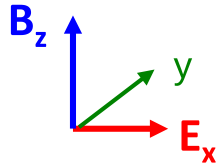
[1]Wong, Phys. Fluids, 13 757 (1970)
[2]Gary and Sanderson J. Plasma Phys. 4 739 (1970)
[3]Forslund et al., Phys. rev. Lett. 25 1266 (1970)
[4]Lampe et al., Phys. Rev. Lett. 26 1221 (1971), Phys. Fluids, 15 662 (1972)
[5]Adam et al., Phys, Plasmas 11 295 (2004)
[7]Cavalier et al., Phys, Plasmas 20 082107 (2013)
[8] Lafleur et al., Phys. Plasmas 23 053503 (2016)

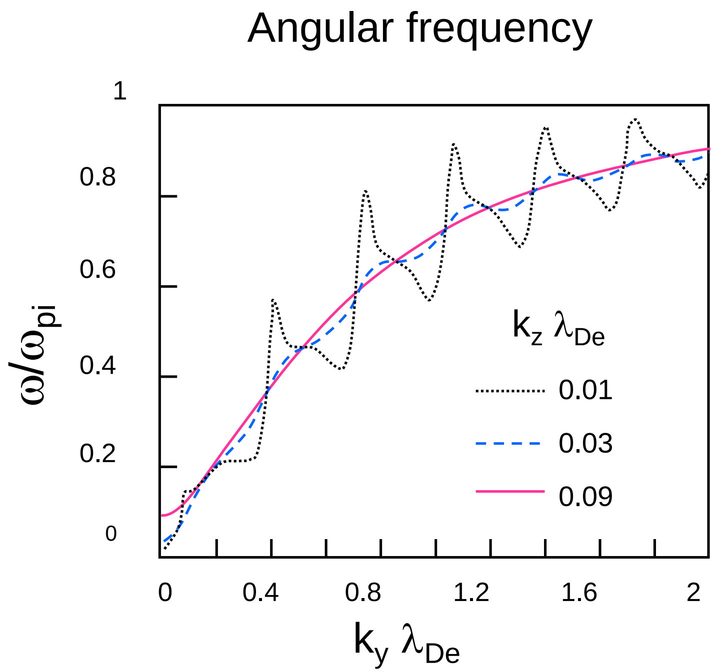
2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices

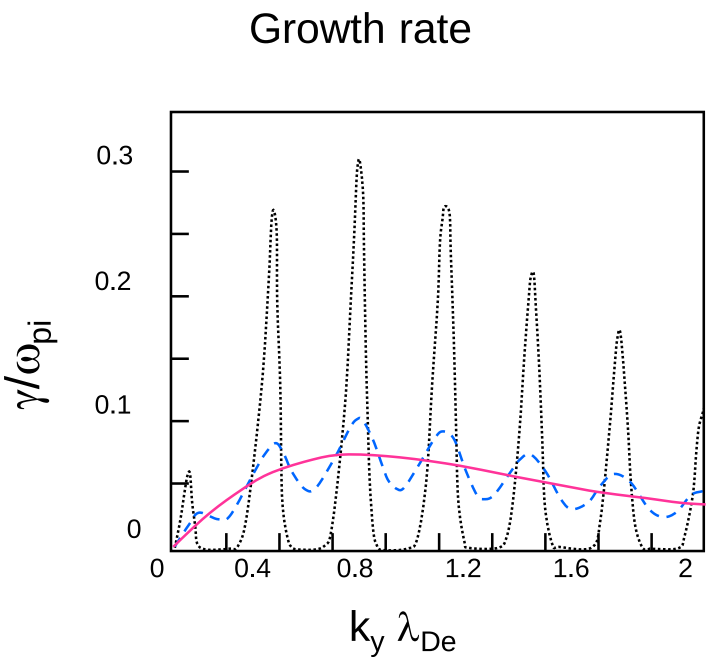
Theory of $\boldsymbol{E} \times \boldsymbol{B}$ Electron Cyclotron Drift Instability

- Solutions of 3D dispersion relation[1] calculated for n=$2\text{x}10^{17}$ m^{-3}, T_e=25 eV, $V_{i,b}$=16 km/s

λ_{De} =$8.3\text{x}10^{-5}$ m, ω_{pi} =$5.1\text{x}10^{7}$ rd/s, V_d/v_{the}=0.3 , Ω_{ce}/ω_{pi}=50







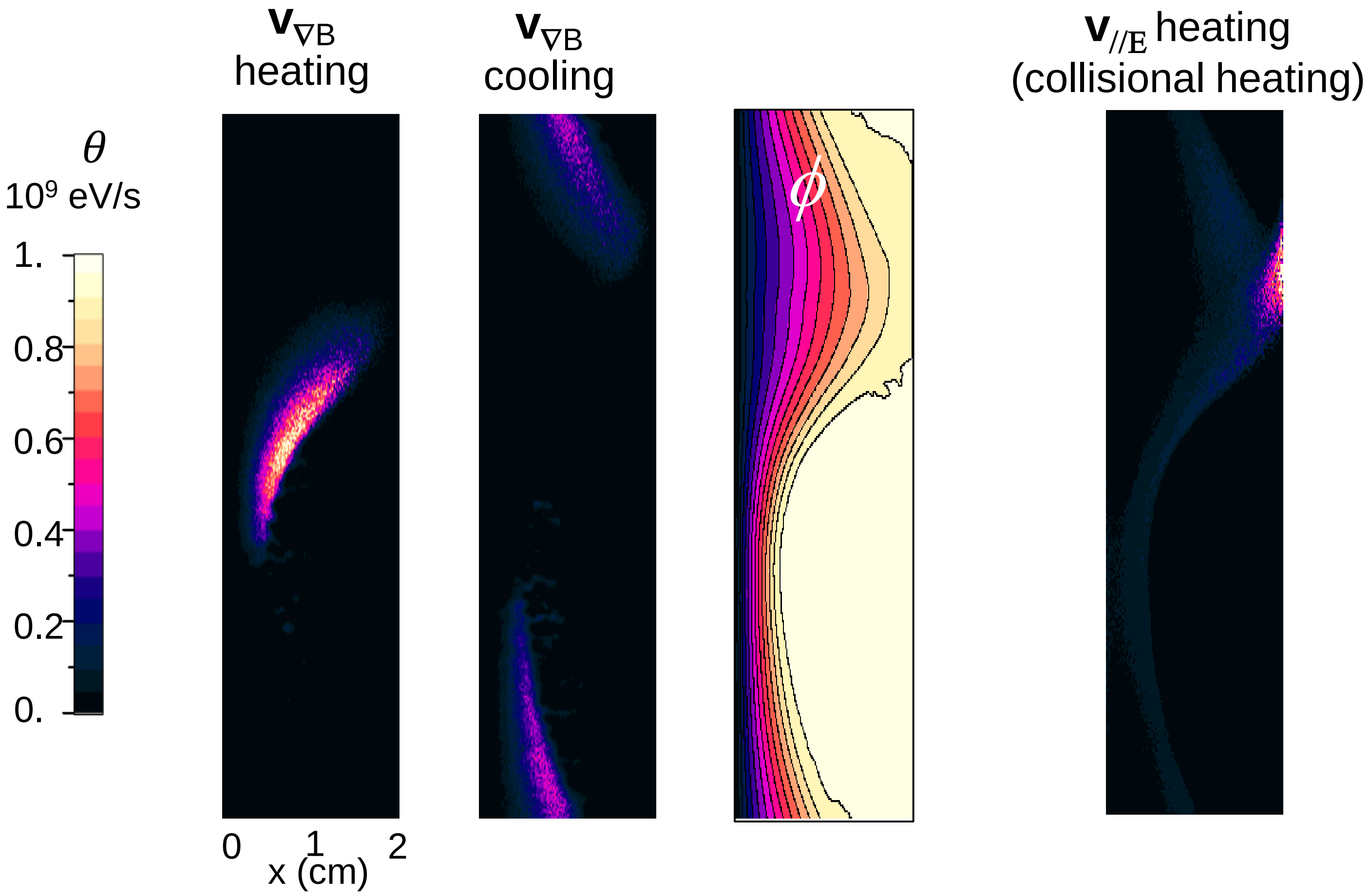
resonances at $k_y \rho_e = n$ for $k_z = 0$

[1] Cavalier et al., Physics of Plasmas 20 082107 (2013)

2. Rotating instabilities in $E \times B$ discharge devices

- PIC-MCC simulation of magnetron discharge

Mechanisms of electron heating



2. Rotating instabilities in $\boldsymbol{E} \times \boldsymbol{B}$ discharge devices – Two examples

➢ Instabilities in a Hall thruster

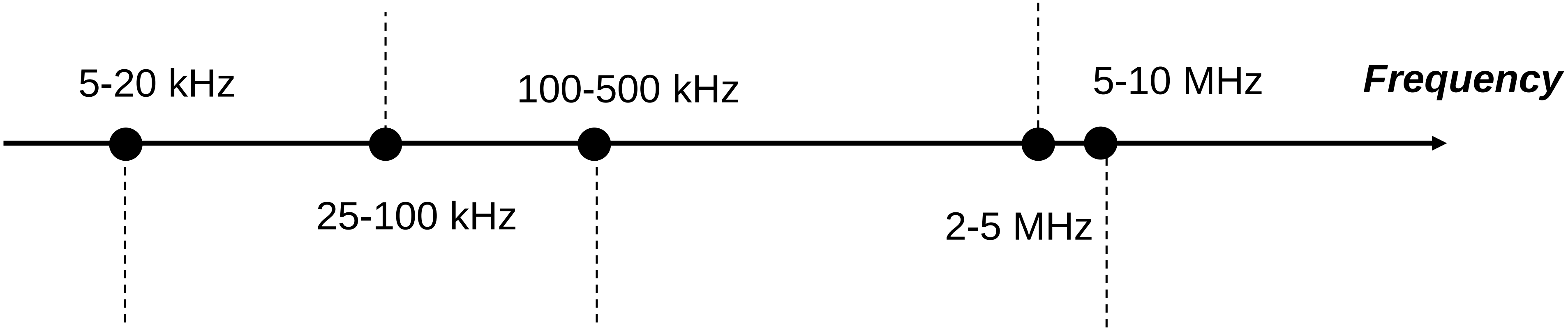
After T Charoy (2021)

Rotating Spokes

Rotation of azimuthal non-uniformities ~ several cms
Triggered by gradient drift instability in $\boldsymbol{E}.\boldsymbol{\nabla n} > \boldsymbol{0}$ region

Electron Cyclotron Drift Instability

Azimuthal oscillations λ ~1 mm
In collisionless acceleration region (where $\boldsymbol{E}.\boldsymbol{\nabla n} < \boldsymbol{0}$)
Appears in axial-azimuthal PIC simulations



Breathing-mode

Strong axial oscillations
λ ~1 cm
Large fluctuations of densities and current
Due to neutral depletion

Ion Transit Time Instability

Axial oscillations λ ~1 cm
Ion wave-riding
Resistive instability

Modified Two-Stream Instability

Azimuthal oscillations λ ~1 cm
In collisionless acceleration region
Coupled with ECDI in radial-azimuthal PIC simulations

More on instabilities in Hall thrusters in next talk by Ben Jorns